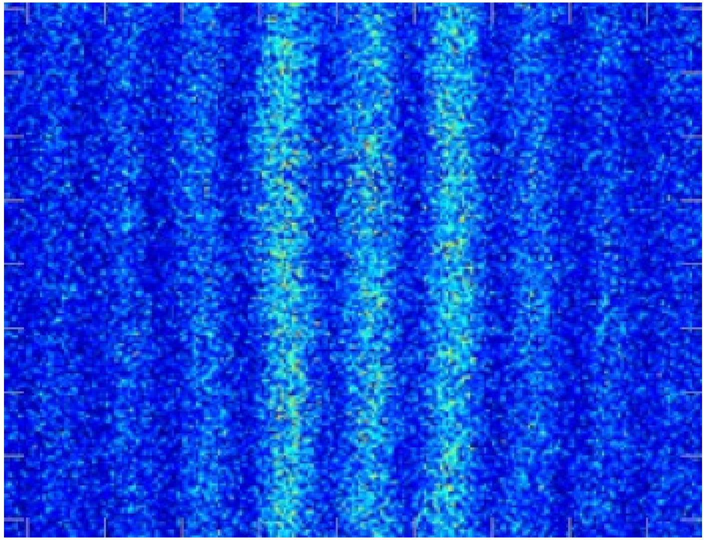
Surface Losses and Decoherence in Electron Interferometry



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March 2026

Outline

- Review electron interferometry experiments
- Look at the logical structure of calculations
- Extend predictions to match experimental results
- Quantify various contributions to decoherence

Electron Inteferometry Experiments

- The classic we all teach: electron fringing as the quantum-mechanical analogue to light diffraction

Electron Diffraction at Multiple Slits

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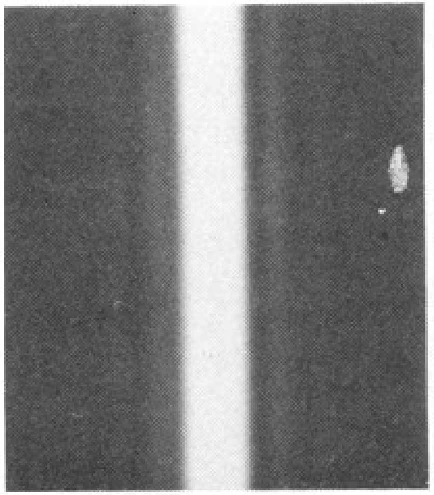
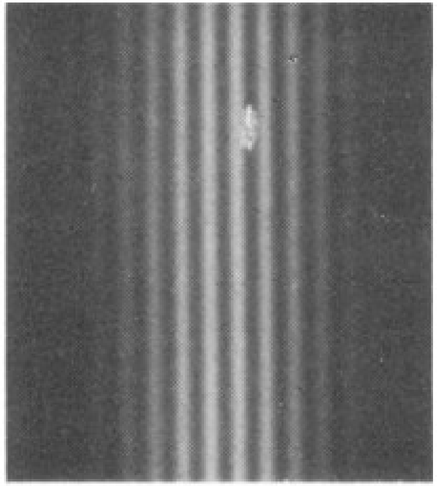
Am. J. Phys. 42, 4–11 (1974)
https://doi.org/10.1119/1.1987592

Single Slit
(40 keV)



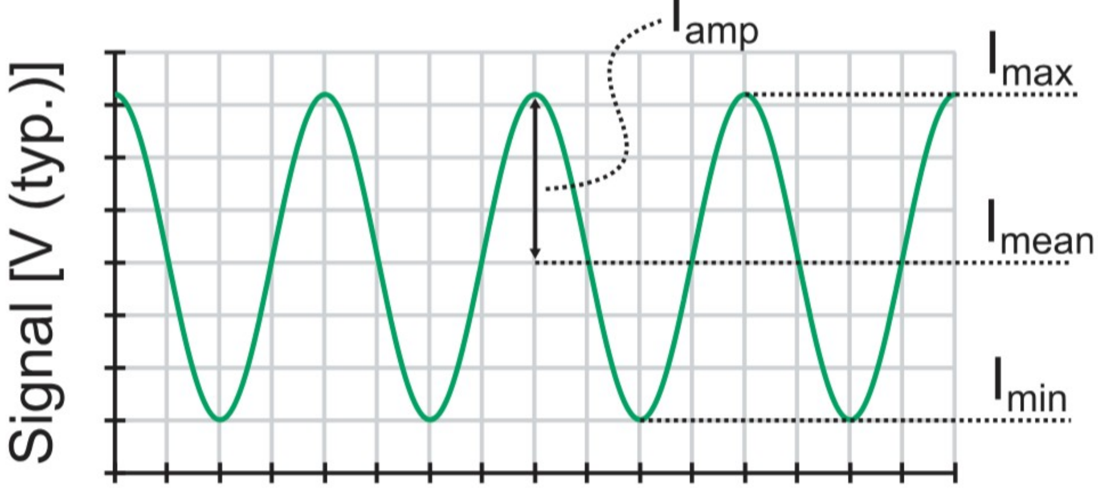
Double Slit
(40 keV)



Fringe Contrast and "Visibility"

- An easy thing to measure: minima and maxima in inteferogram
- Allows us to find the fringe visibility:

$$\mathcal{V} = \frac{I_{\rm max} - I_{\rm min}}{I_{\rm max} + I_{\rm min}} = \frac{I_{\rm amp}}{I_{\rm mean}}$$



- Limits: $V = 1 \rightarrow$ max contrast
  $V = 0 \rightarrow$ min contrast

Experiments in Fringe Visibility

- Empirical observation: fringe visibility decreases as the electron
  - Passes nearer to the dielectric
  - Has farther-separated paths

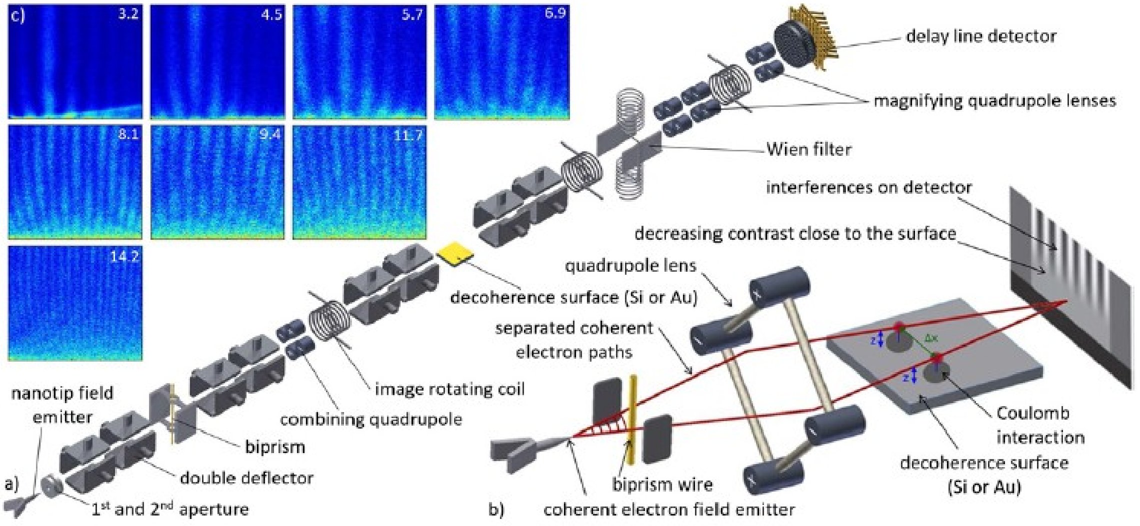


Figure 1. (a) Experimental setup for electron decoherence measurements close to a semiconducting and metallic environment. (b) Sketch of the experiment, where a coherent electron beam is separated and combined by an electrostatic biprism and a quadrupole lens. In the quantum superposition state before interference, the beam passes aloof a semiconducting n-doped silicon or gold surface, interacting with the bulk electron gas. This process causes decoherence which can be observed in a decreasing interference contrast close to the surface. (c) Recorded fringe pattern above the silicon surface (bottom edge of each image) for different beam path separations Δx (value in upper right corner in μm). The top edge is at a height $z = 40$ μm. An increase in signal above the surface up to max 4 μm is visible that is assumed to be due to scattering on waver edge defects.

"Quantum decoherence by Coulomb interaction"
New J. Phys. 22 (2020) 063039

Experiments in Fringe Visibility

- Kerker *et al.* (2020) tested various models of visibility predicted from entanglement of electron and plate

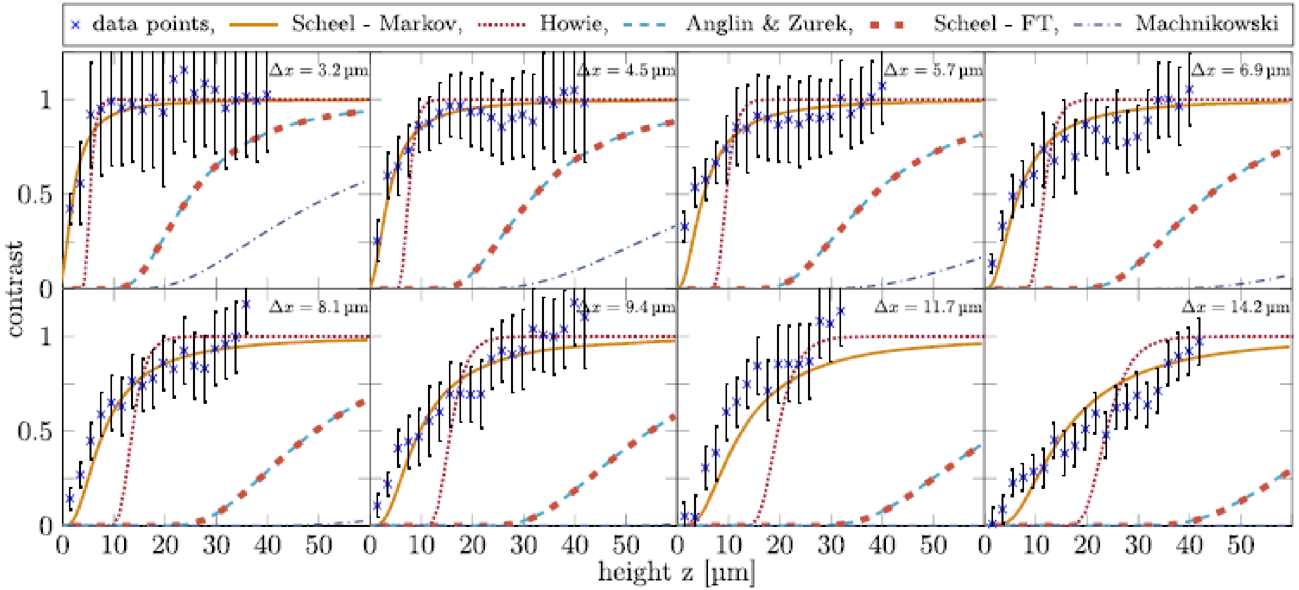
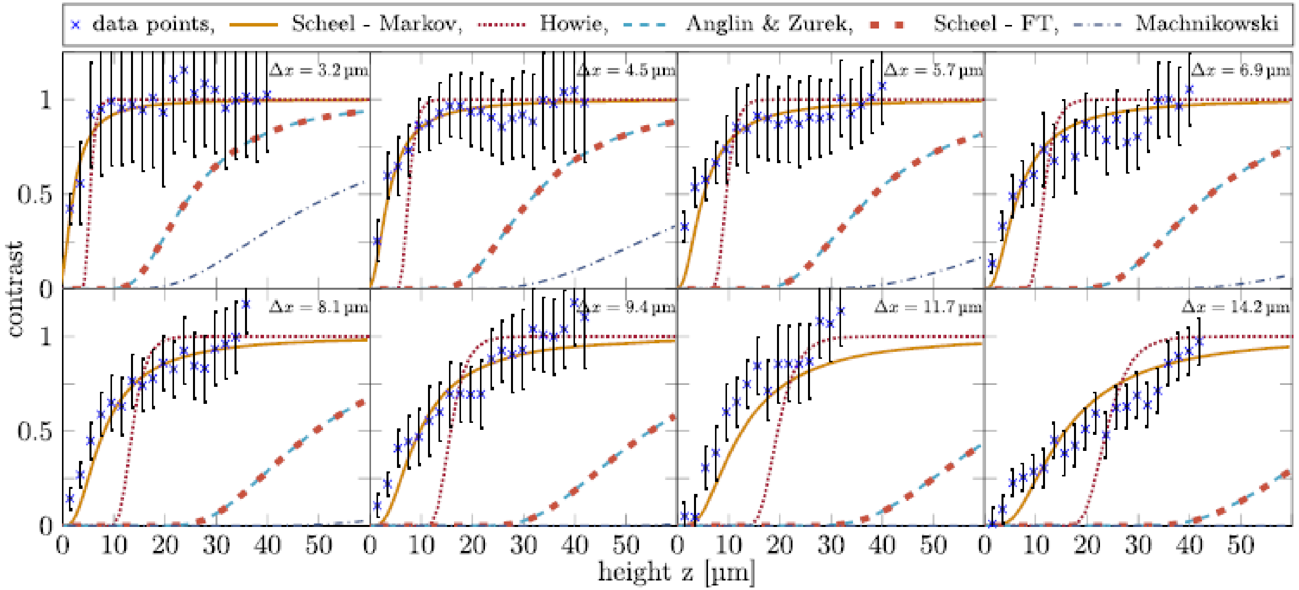


Figure 2. Decoherence of electrons in a superposition state close to a doped silicon surface. The loss of visibility is plotted as a function of distance z to the semiconducting plate for different beam path separations Δx and compared to four current theoretical models [5–8].

Experiments in Fringe Visibility

- Kerker *et al.* (2020) tested various models of visibility predicted from entanglement of electron and plate
- Can the Scheel-Markov model be recovered from wavefunctions, rather than density matrices?



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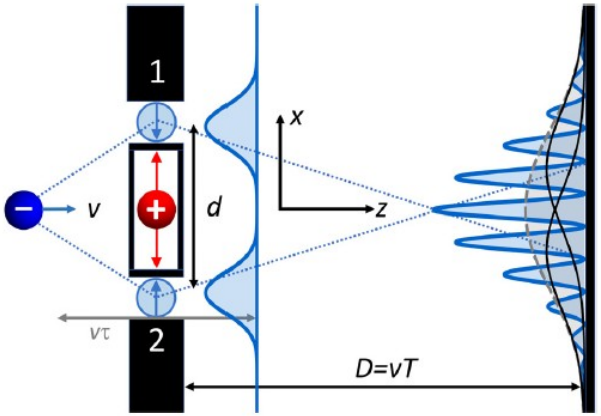
Pedagogical Model of Fringe Visibility

- 1D quantum-mechanical model in Am J. Phys. (2025)
- x = coordinate of propagating electron
- X = coordinate of proton in slit
- Probability of observing electron at position x on screen:

$$P_e(x) = \int_{-\infty}^{\infty} |\Psi_{out}(x, X)|^2 dX.$$



Decoherence, entanglement, and information in the electron double-slit experiment with monitoring

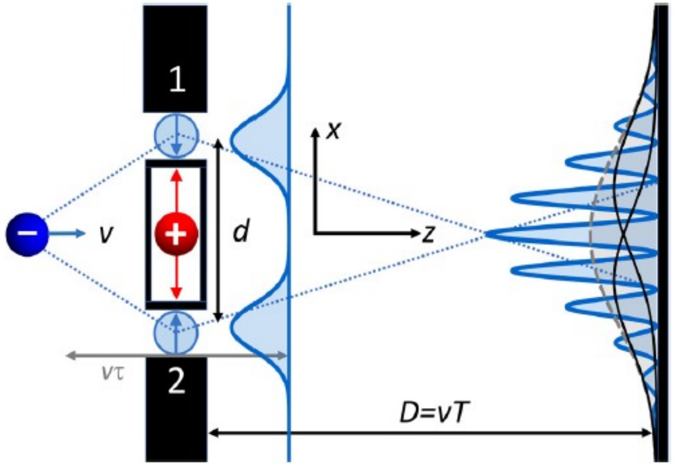
Frederick W. Strauch[a)]

https://doi.org/10.1119/5.0210117

Pedagogical Model of Fringe Visibility

- In Strauch's 1D model, "which path" information is encoded as a phase factors to conserve momentum

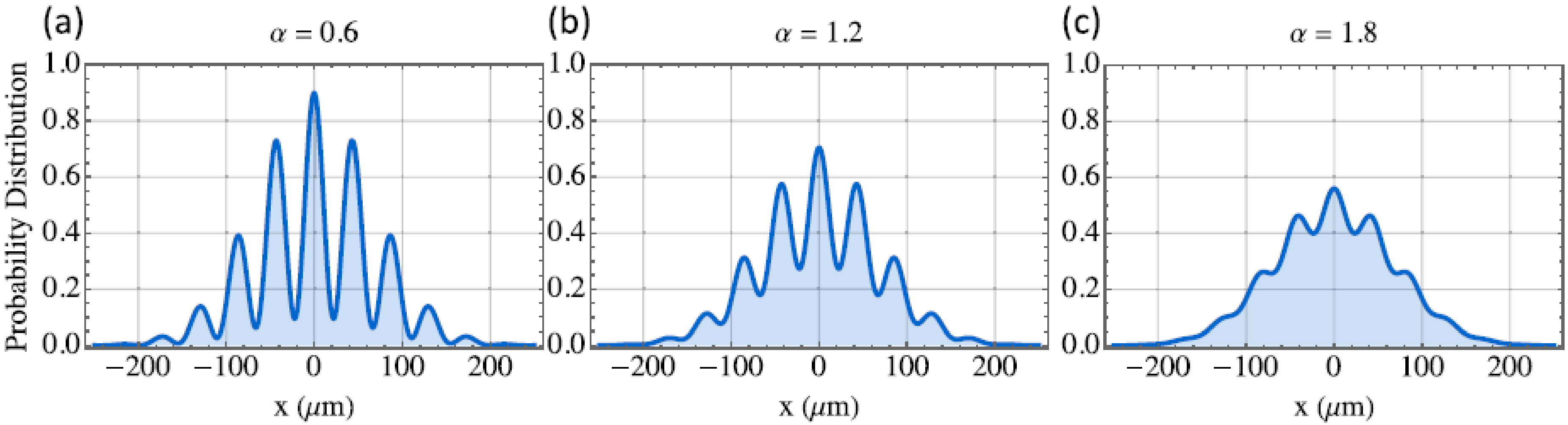
$$\Psi_{\text{in}}(x,X)=\frac{1}{\sqrt{2}}\psi_1(x)\psi_{\text{p}}(X)+\frac{1}{\sqrt{2}}\psi_2(x)\psi_{\text{p}}(X) \longrightarrow \Psi_{\text{out}}(x,X)=\frac{1}{\sqrt{2}}\psi_1(x)e^{-iPx/\hbar}\psi_{\text{p}}(X)e^{+iPX/\hbar}+\frac{1}{\sqrt{2}}\psi_2(x)e^{+iPx/\hbar}\psi_{\text{p}}(X)e^{-iPX/\hbar}.$$



$$\frac{1}{\sqrt{2}}|\text{"electron in slit 1"}\rangle|\text{"proton moving up"}\rangle+\frac{1}{\sqrt{2}}|\text{"electron in slit 2"}\rangle|\text{"proton moving down"}\rangle.$$

Pedagogical Model of Fringe Visibility

- Summing over proton states smears out the observable electron fringes: $P_e(x) = \int_{-\infty}^{\infty} |\Psi_{out}(x, X)|^2 dX.$
- Visibility reduction correlates with interaction



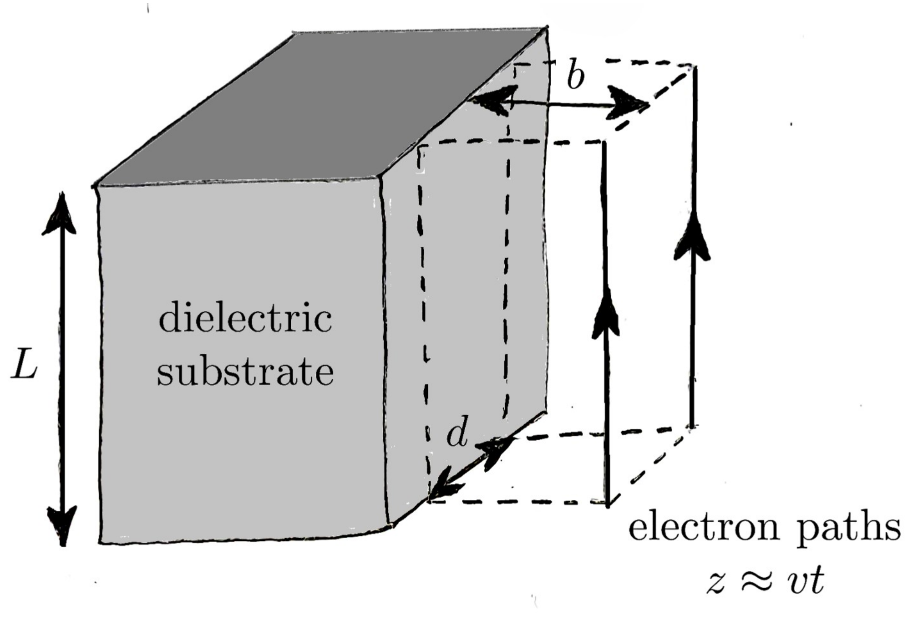
Increasing interaction strength
Decreasing fringe visibility

3D Version of the Interaction Model

- Experimental situation is nearly the same as Strauch, but "which path" information depends on *L*, *d*, and *b*
- Before interaction, assume the substrate is in its ground state, and the electron is in a split-path

$$|\Psi_e\rangle = \frac{1}{\sqrt{2}}[\psi_{\mathrm{R}}(\mathbf{x}_\perp) + \psi_{\mathrm{L}}(\mathbf{x}_\perp)]$$

$$|\Psi_i\rangle = |\Psi_e\rangle \otimes |0\rangle \, .$$



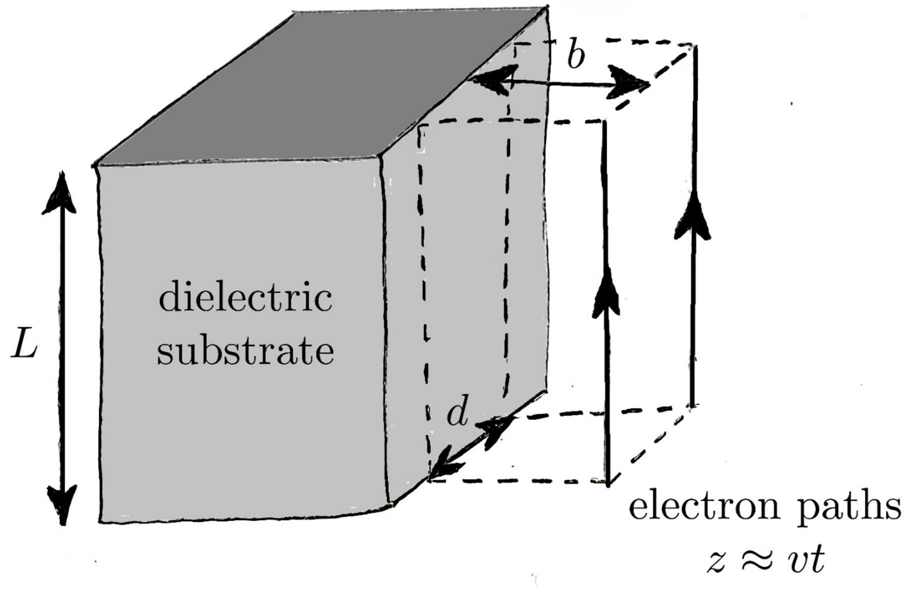
3D Version of the Interaction Model

- Keeping first-order excitations of surface modes with wavevector **k**, the wavefunction after interaction is

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}}\left[\psi_{\mathrm{R}}(\mathbf{x}_\perp)\left(c_{\mathrm{R}0}\,|0\rangle + \sum_{\mathbf{k}} c_{\mathrm{R}\mathbf{k}}\,|1_{\mathbf{k}}\rangle\right)\right.$$
$$\left. + \psi_{\mathrm{L}}(\mathbf{x}_\perp)\left(c_{\mathrm{L}0}\,|0\rangle + \sum_{\mathbf{k}} c_{\mathrm{L}\mathbf{k}}\,|1_{\mathbf{k}}\rangle\right)\right]$$

- Afterward, the wave packets are made to overlap, and we sum over surface modes:

$$P_e(\mathbf{x}_\perp) = \sum_{\{n_{\mathbf{k}}\}} \langle\Psi_f|\mathbb{I}_{\mathrm{el}} \otimes |\{n_{\mathbf{k}}\}\rangle\,\langle\{n_{\mathbf{k}}\}|\,|\Psi_f\rangle$$



3D Version of the Interaction Model

- If paths are equally weighted, we can use symmetry conditions—e.g., $\psi_{\mathrm{R}}(\mathbf{x}_\perp) = \psi^*_{\mathrm{L}}(\mathbf{x}_\perp) = \psi_0(\mathbf{x}_\perp)$
- Such symmetry conditions allow us to reduce

$$P_e(\mathbf{x}_\perp) = |\psi_0(\mathbf{x}_\perp)|^2 + \mathrm{Re}\left\{ (\psi_0(\mathbf{x}_\perp))^2 \left[c^*_{\mathrm{R}0} c_{\mathrm{L}0} + \sum_{\mathbf{k}} c^*_{\mathrm{R}\mathbf{k}} c_{\mathrm{L}\mathbf{k}} \right] \right\}$$

to calculate the visibility via scattering coefficients:

$$\mathcal{V} = 1 - \sum_{\mathbf{k}} |c_{\mathbf{k}}|^2 [1 - \cos(k_x d)].$$

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$\mathcal{V}$

to calculate the visibility via scattering coefficients:

$$\mathcal{V} = 1 - \sum_{\mathbf{k}} |c_{\mathbf{k}}|^2 [1 - \cos(k_x d)].$$

Total scattering probability….weighted

Markovian Rate Model

- For quantized surface modes, the calculated scattering probability scales with the length *L*

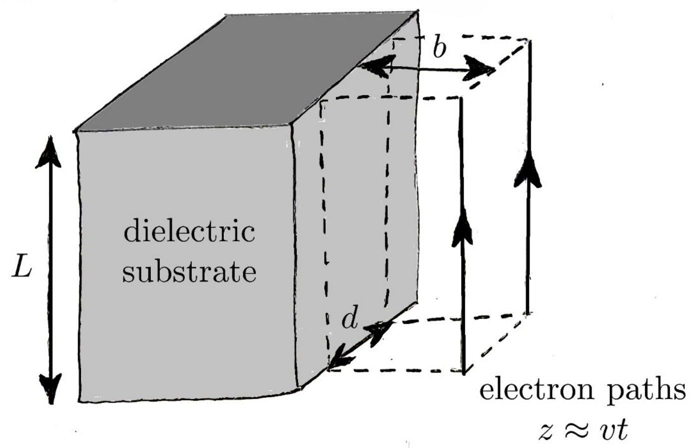
$$|c_{0\to 1_{\mathbf{k}}}|^2 = L\,\frac{dP_{\mathbf{k}}}{dz}$$

$$V(L) \approx 1 - L\sum_{\mathbf{k}} \frac{dP_{\mathbf{k}}}{dz}\left[1 - \cos(k_x d)\right]$$

- Reinterpreting this as a continuous process, we obtain

$$\gamma(b,d) = \sum_{\mathbf{k}} \frac{dP_{\mathbf{k}}}{dz}\left[1 - \cos(k_x d)\right]$$

$$\boxed{\mathcal{V}(b,d,L) = \exp[-\gamma(b,d)L].}$$



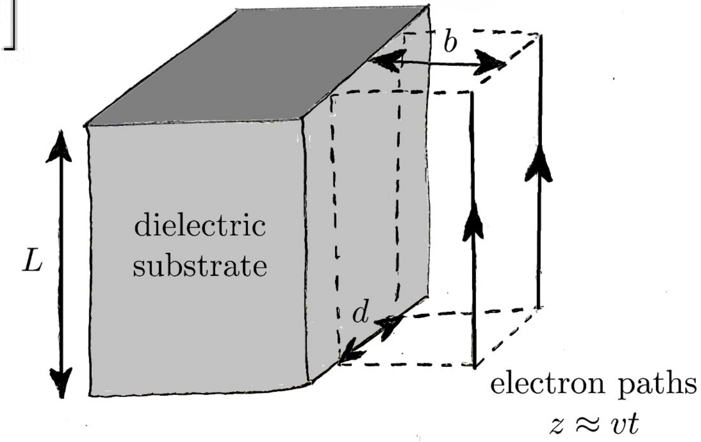
Generalizing 3D Interaction Model

- Electron energy-loss spectroscopy community has models for loss-per-mode-per-unit length, which lets us extend this ``weighted probability” picture
- If we consider our expressions as rates, we can write

$$\gamma(b,d) = \int_0^\infty d\omega \int_{-\infty}^{\infty} dk_x \, \frac{d^3P}{d\omega\, dk_x\, dz} \left[1 - \cos(k_x d)\right]$$

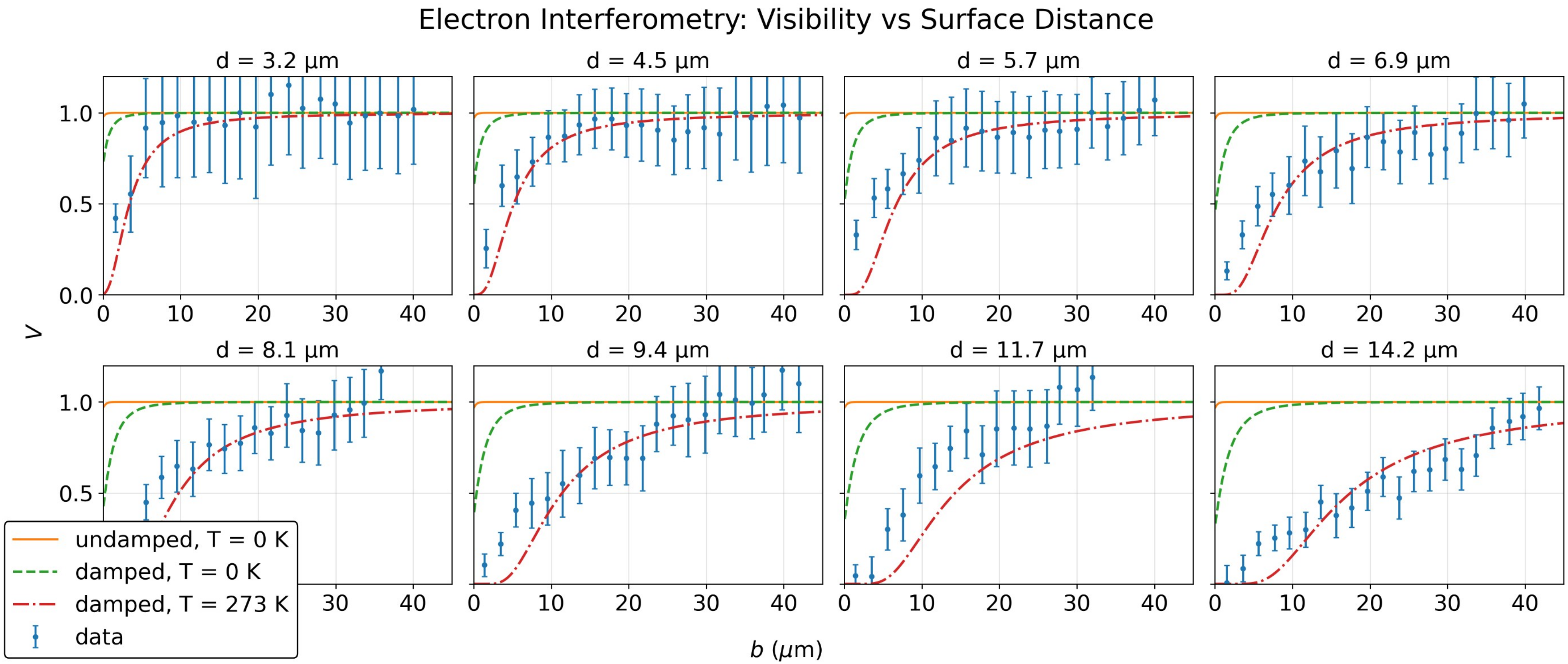
with the visibility evolving as

$$\mathcal{V}(b,d,L) = \exp[-\gamma(b,d)L]$$



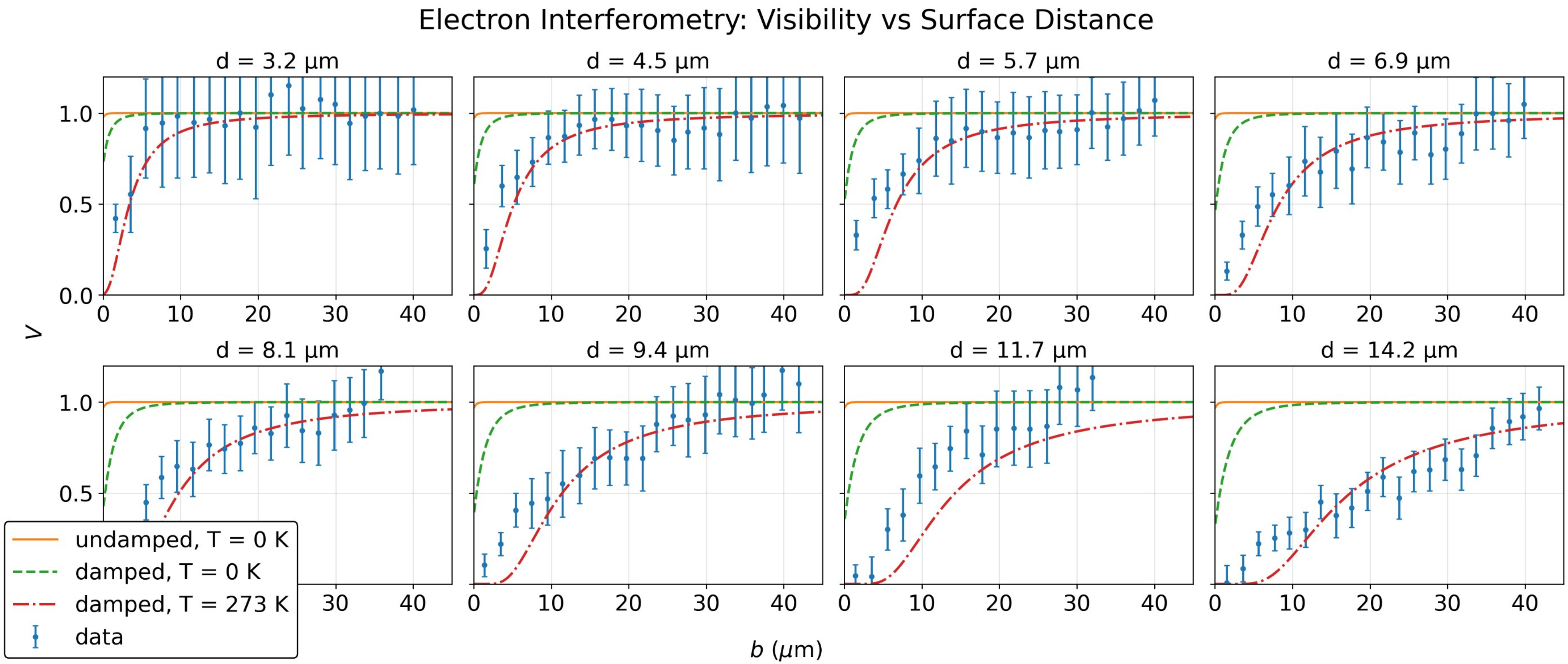
Pinpointing Contributions to Decoherence

- Models for $d^3P/d\omega\, dk_x\, dz$ can incorporate mode damping and thermal effects—both are important!



Pinpointing Contributions to Decoherence

- For the 1 keV beam in this experiment, retardation effects are not important, but could be in principle



Conclusion

- Electron inteferometry can be described using standard wavefunction methods, in the limit of continuous processes
- For past experiments, thermal and mode damping effects dominate—can future experiments seek other possibilities?