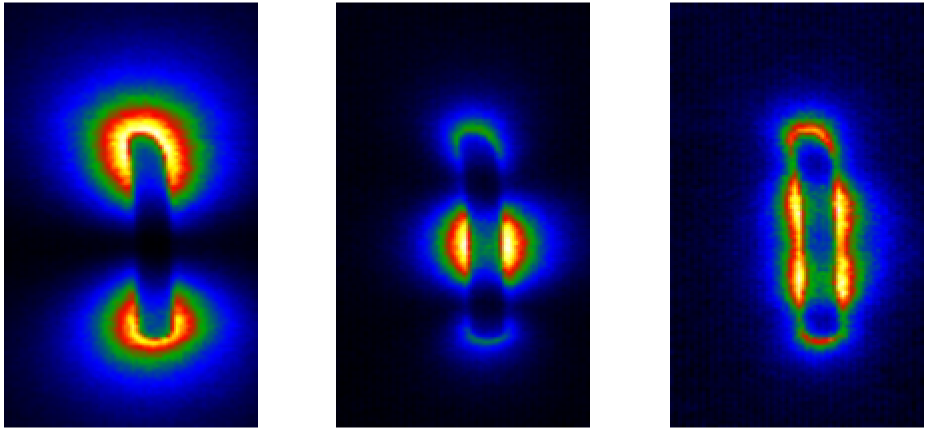
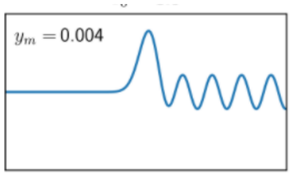
Adventures with Oscillators!

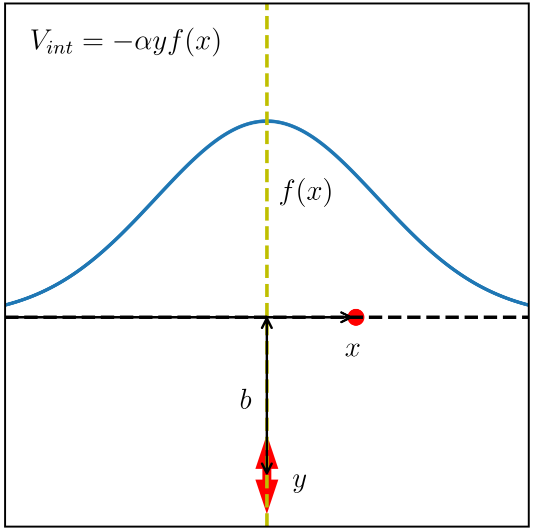
Beer Bottles, Nanorods, and a Classical/Quantum Comparison

Dr. David Kordahl

January 31, 2025





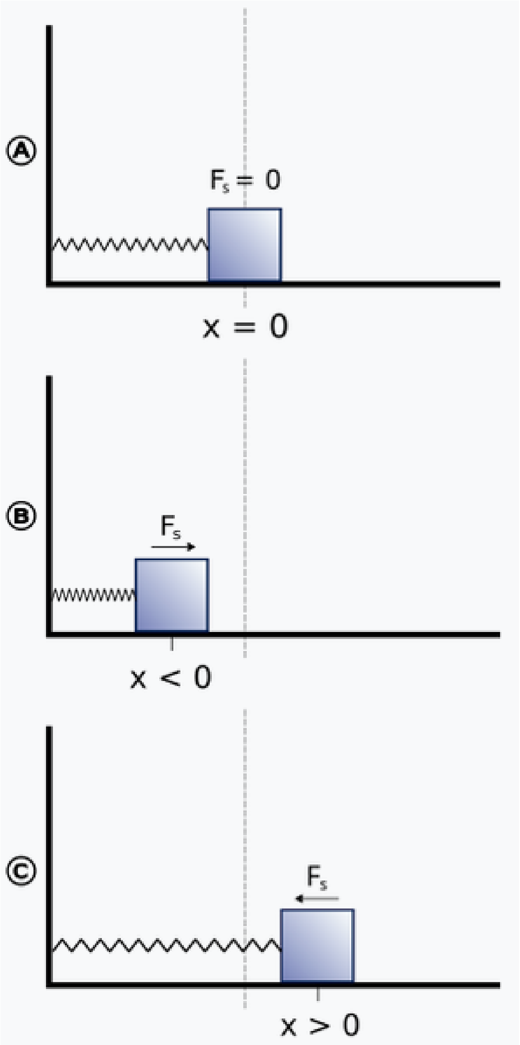


Outline

- **Physical models often apply across contexts**
  - An old favorite: the driven (damped) oscillator
- **Applications of the driven oscillator**
  - Classical: acoustical resonance of a beer bottle
  - Quantum: plasmonic resonance of a nanoparticle
- **Compare/contrast classical + quantum problems**
  - Look at simplified oscillator problem

A favorite toy

- The harmonic oscillator maps onto many different physical systems that oscillate



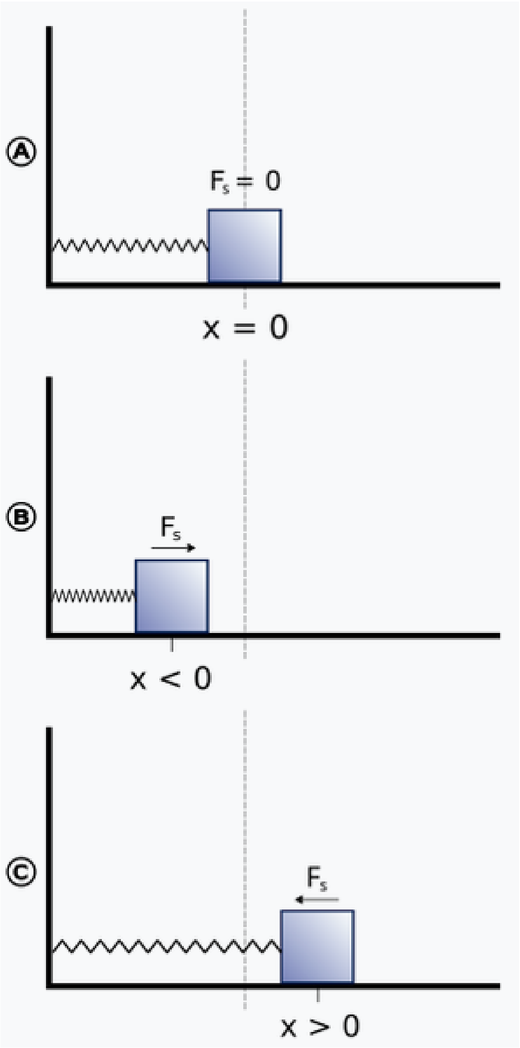
From the Wikipedia article on the HOs

A favorite toy

- **The harmonic oscillator maps onto many different physical systems that oscillate**

- **Two "tunable parameters"**
  - Block mass ***m***
  - Spring stiffness ***k***

$$\ddot{x} = -\frac{k}{m}x$$



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A favorite toy

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- **Two "tunable parameters"**
  - Block mass ***m***
  - Spring stiffness ***k***

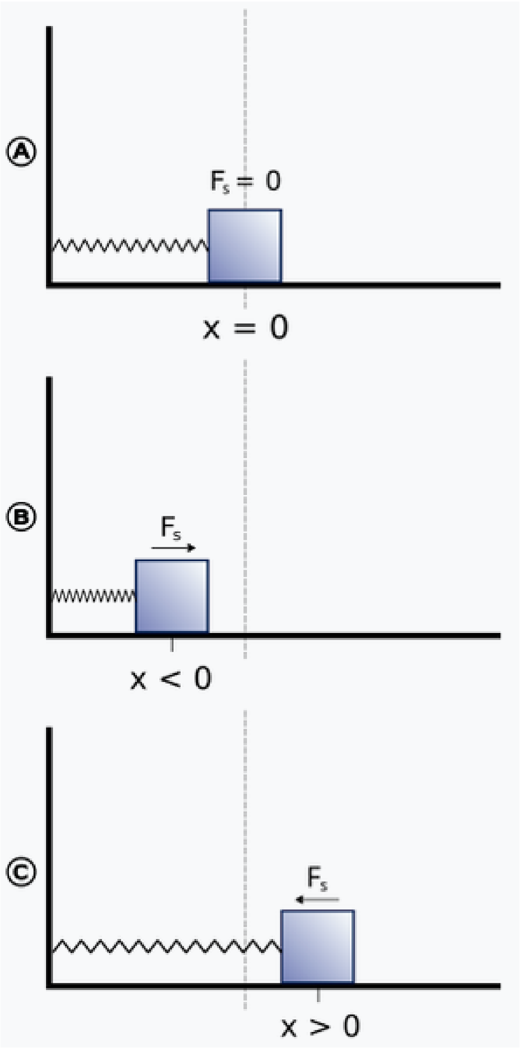
$$\ddot{x} = -\frac{k}{m}x$$

- **These variables control oscillation frequency**

$$x = x_0 \cos(\omega_0 t - \delta)$$

$$\ddot{x} = \omega_0^2 x$$

$$\rightarrow \omega_0^2 = \frac{k}{m}$$



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Engineering some reality

- The obvious next step: **add a damping term**

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

Engineering some reality

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- Taylor's "Classical Mechanics" gives **solution** as

$$x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta).$$

$$\omega_1 = \sqrt{\omega_o^2 - \beta^2}.$$

$$\text{(decay parameter)} = \beta$$

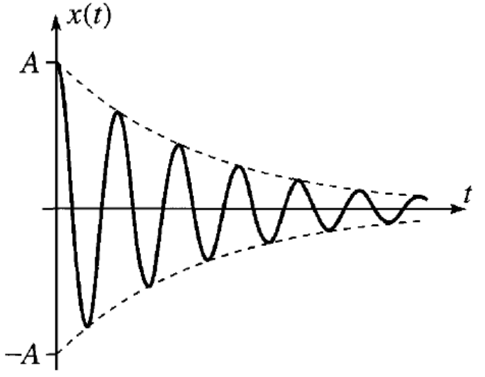

Figure 5.11 Underdamped oscillations can be thought of as simple harmonic oscillations with an exponentially decreasing amplitude $Ae^{-\beta t}$. The dashed curves are the envelopes, $\pm Ae^{-\beta t}$.

Adding external force

- One more step: **add an external force**

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F(t)/m$$

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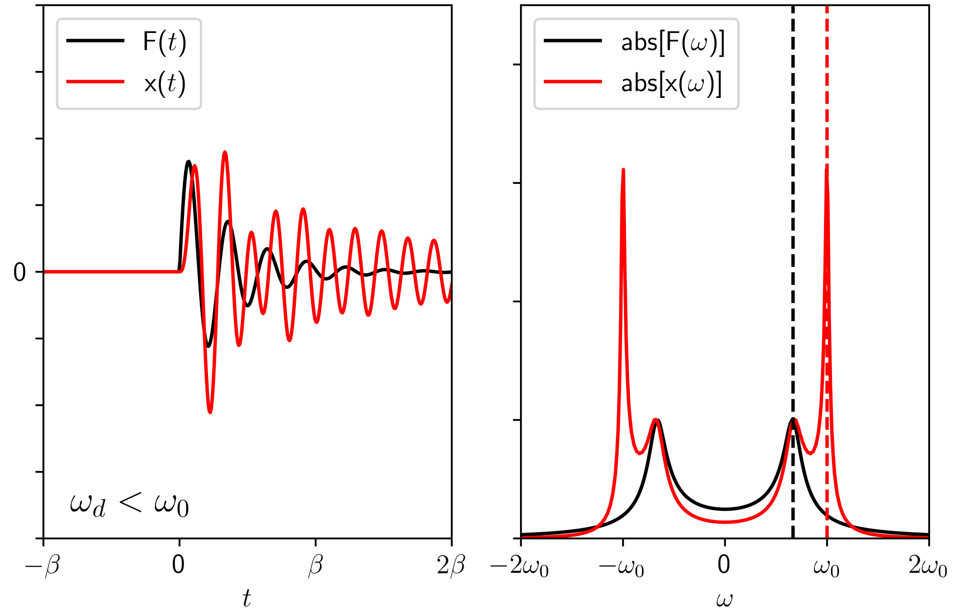

Fourier transform of force and oscillations

$$\mathscr{F}\left[f(t)\right] = \tilde{f}(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt$$

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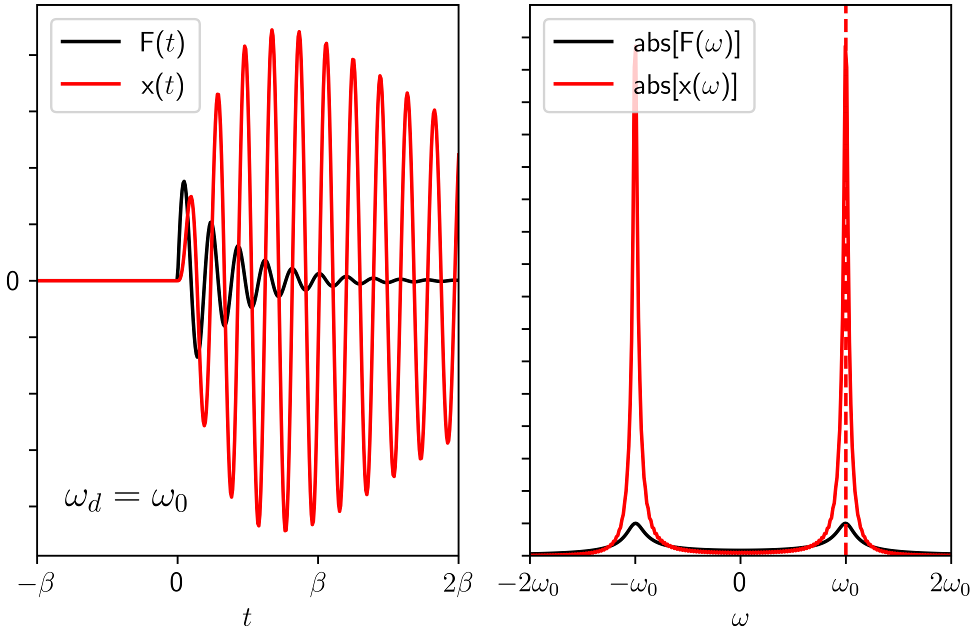

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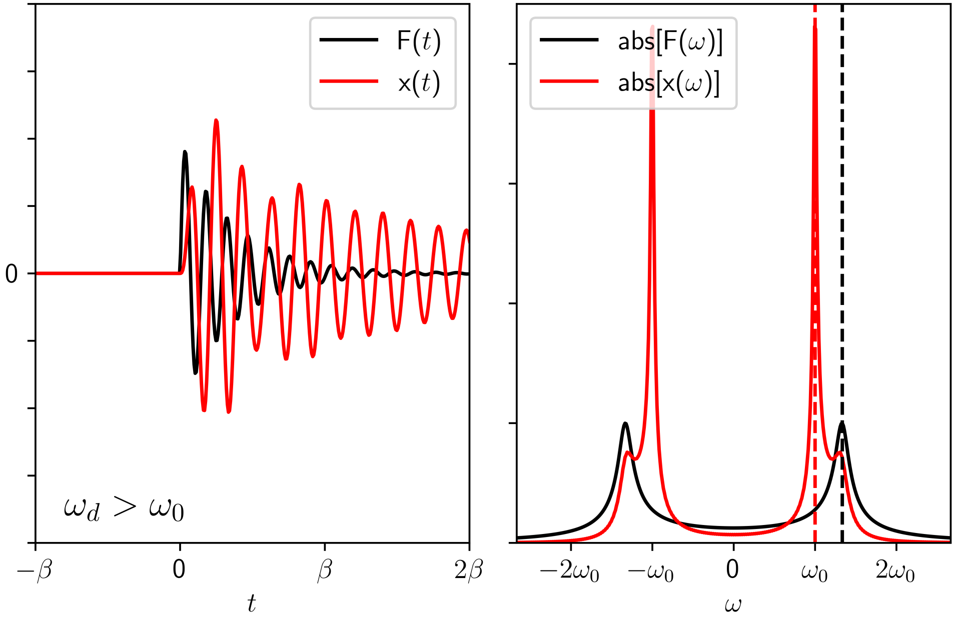

Fourier transform of force and oscillations

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Green's functions

- We want to **connect solutions**

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- One method: we can use a **Green's function**!

$$x(t) = \int_{-\infty}^{+\infty} G(t-t')F(t')dt'.$$

Green’s functions

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- One method: we can use a Green’s function!

$$x(t) = \int_{-\infty}^{+\infty} G(t-t')F(t')dt'.$$

- Fourier transforms turn **convolutions** into **products**:

$$\tilde{x}(\omega) = \tilde{G}(\omega)\tilde{F}(\omega).$$

$$\tilde{G}(\omega) = \frac{-1/m}{(\omega^2 - \omega_0^2) - 2i\beta\omega}$$

Green's function of the DDO

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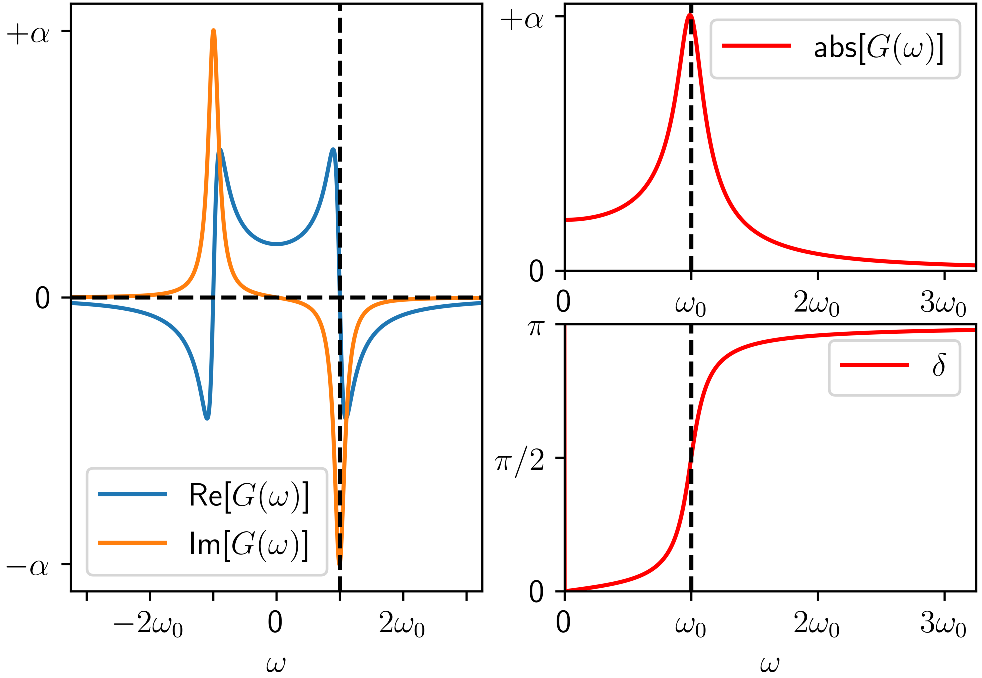
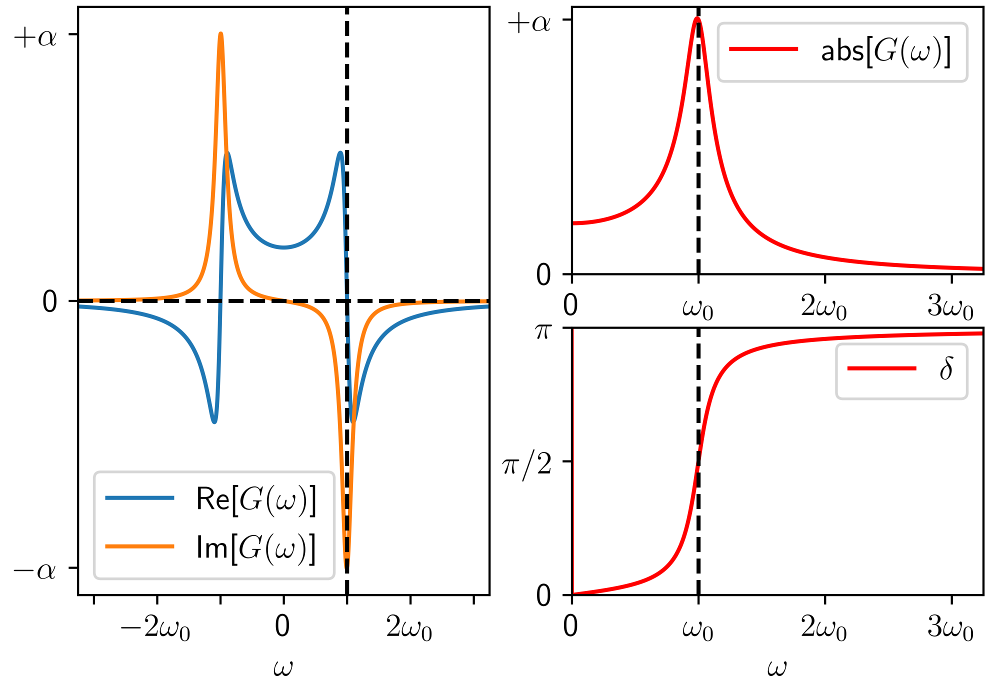


Observations re: the DDO Green's function

- **Amplitude greatest near ω_0**
- $G(\omega)$ produces position oscillations ***in phase*** with the force at **low frequency**
- $G(\omega)$ produces position oscillations ***out of phase*** with force at **high frequency**



$$\tilde{x}(\omega) = \tilde{G}(\omega)\tilde{F}(\omega).$$

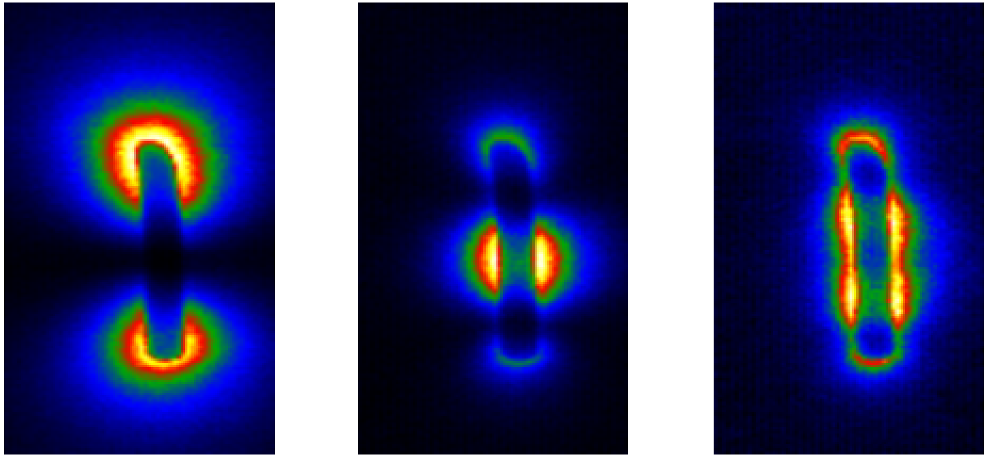
$$\tilde{G}(\omega) = \frac{-1/m}{(\omega^2 - \omega_0^2) - 2i\beta\omega}$$

Models across contexts

- The driven (damped) oscillator can help us understand phenomena that span from classical to quantum

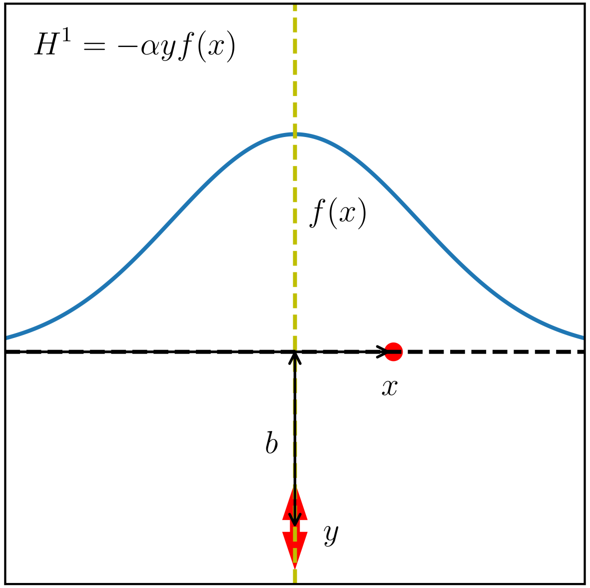
Classical: beer bottle acoustics

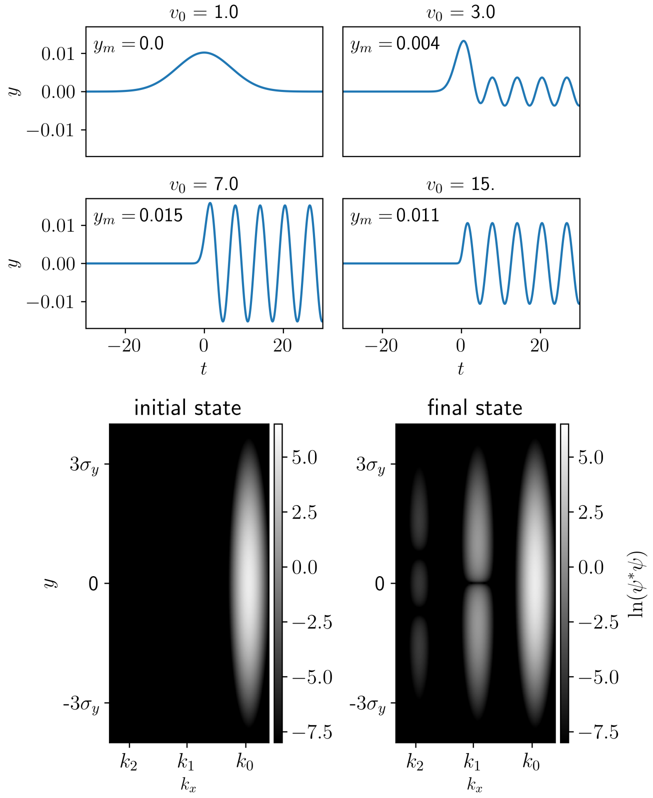
Quantum: nanorod plamons



Abstraction preview…

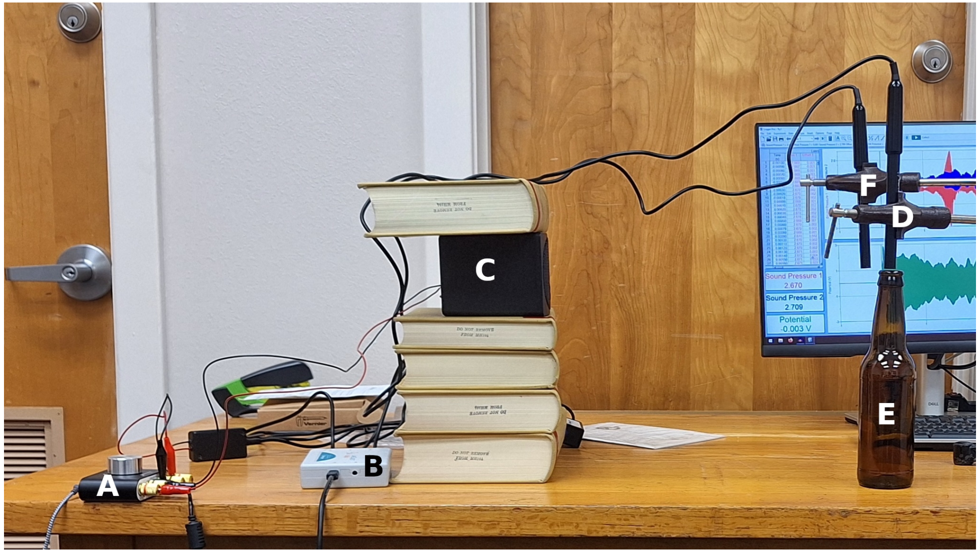
- I'll end the talk with a simple toy model, contrasting classical and quantum approaches

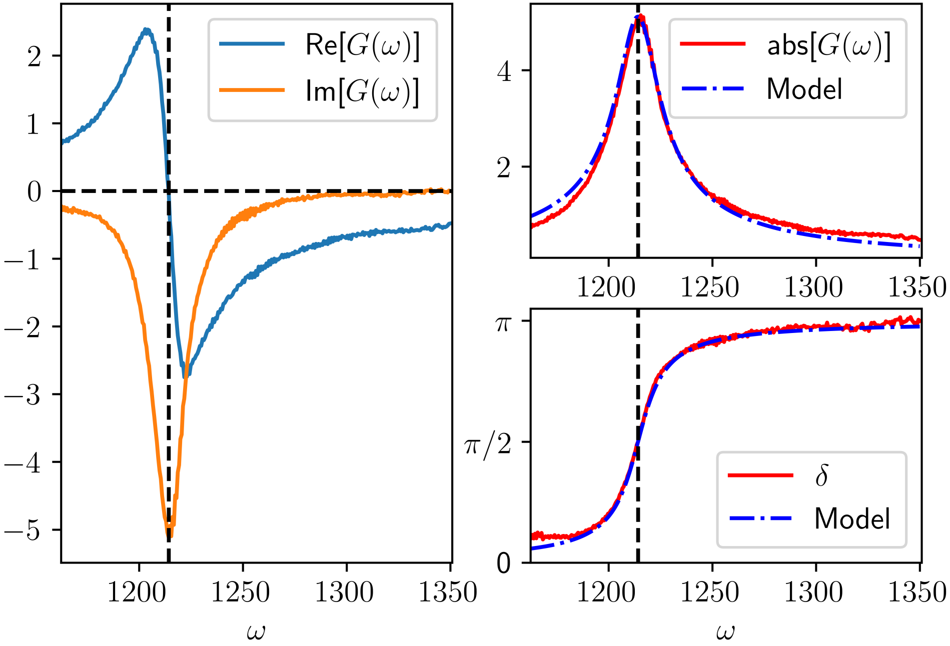




Project 1: Measuring the acoustical Green's function of a beer bottle

- **Pressure oscillations** above a beer bottle **act like DDOs**
- Could we **measure** their associated **Green's function**?





Background reading

An undergraduate experiment demonstrating the physics of metamaterials with acoustic waves and soda cans

James T. Wilkinson; Christopher B. Whitehouse; Rupert F. Oulton; Sylvain D. Gennaro

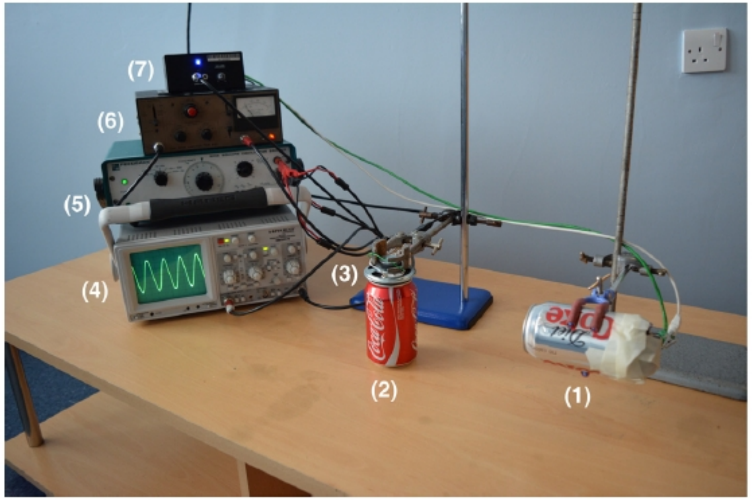
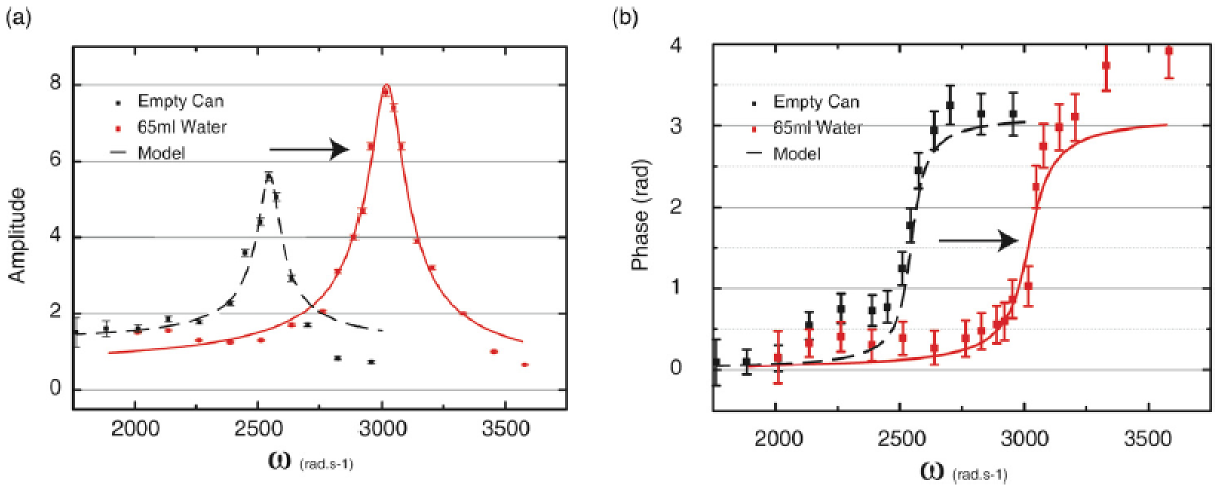


Fig. 2. Experimental apparatus: (1) source (can plus loudspeaker); (2) can under investigation; (3) receiver (loudspeaker used as microphone); (4) oscilloscope; (5) frequency generator; (6) microphone amplifier; (7) source amplifier.



Helmholtz resonators

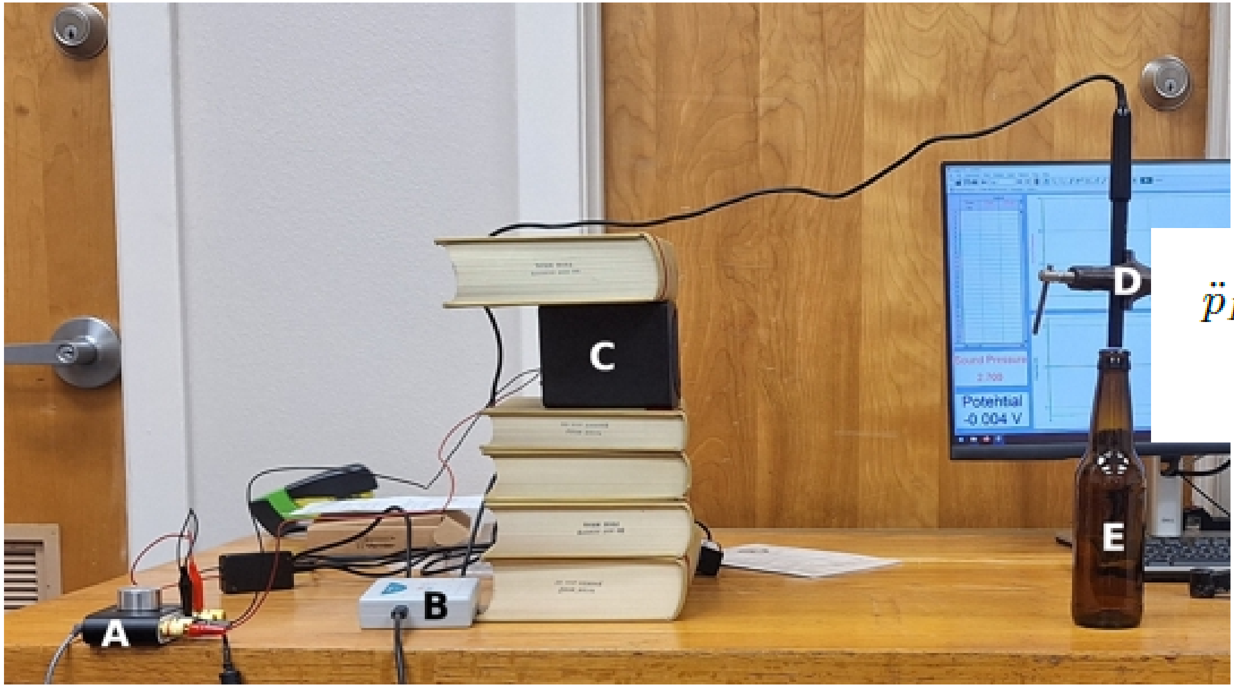
- Imagine an **oscillating plug of air** in the resonator's opening
- The **force** on the air plug depends on…
  - **area** of the opening
  - internal **volume** of the resonator
- This yields a prediction for ω_0:

$$\omega_0^2 \approx \frac{\gamma P_A A^2}{m V_0}.$$

https://en.wikipedia.org/wiki/Helmholtz_resonance

Helmholtz resonators as DDOs

- *Even if* the beer bottle does not act as an ideal Helmholtz resonator, the damped driven oscillator should basically apply



- Introduces resonance, damping, and coupling parameters

$$\ddot{p}_B(t) = -\underbrace{\omega_0^2 p_B(t)}_{\text{restoring force}} + \underbrace{2\alpha\beta\omega_0 p_S(t)}_{\text{driving force}} - \underbrace{2\beta\dot{p}_B(t)}_{\text{damping force}}\,.$$

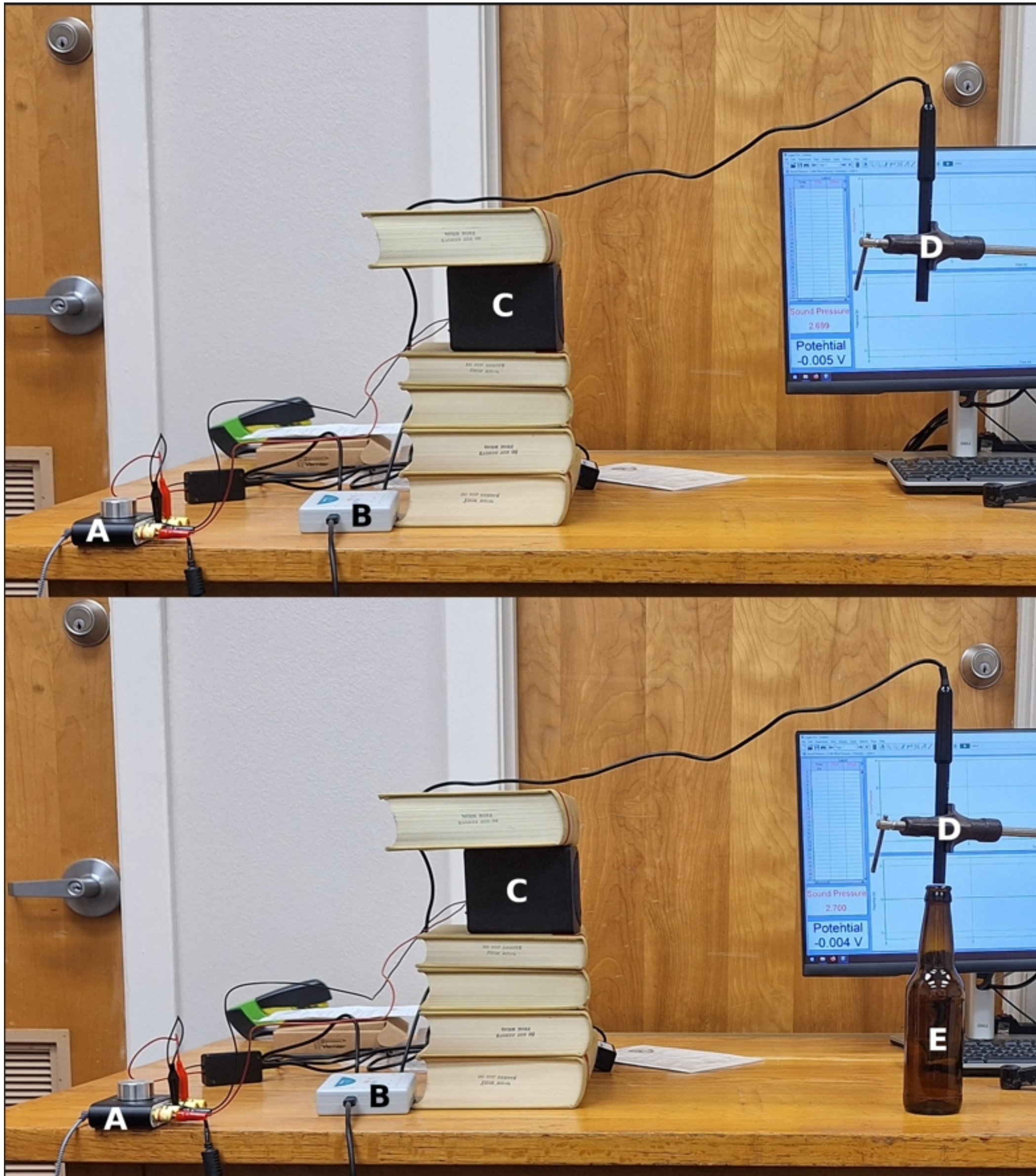
Setup

- First, record sound *without* the bottle:

$$\underbrace{p_S(t)}_{\text{speaker}}$$

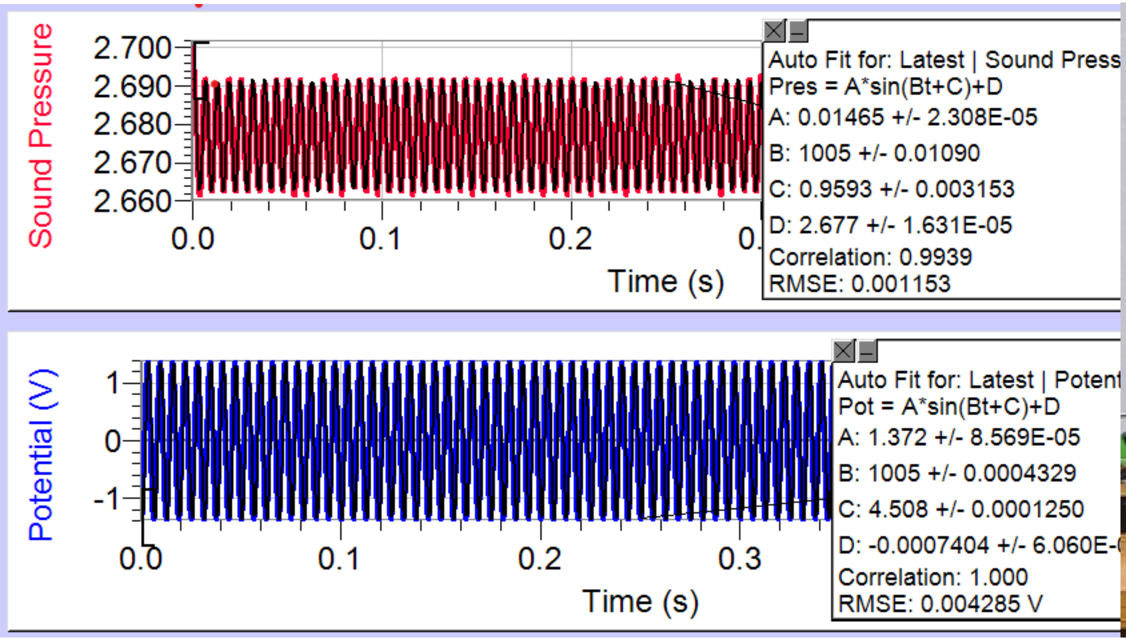
- Next, record sound *with* the bottle to find its contribution:

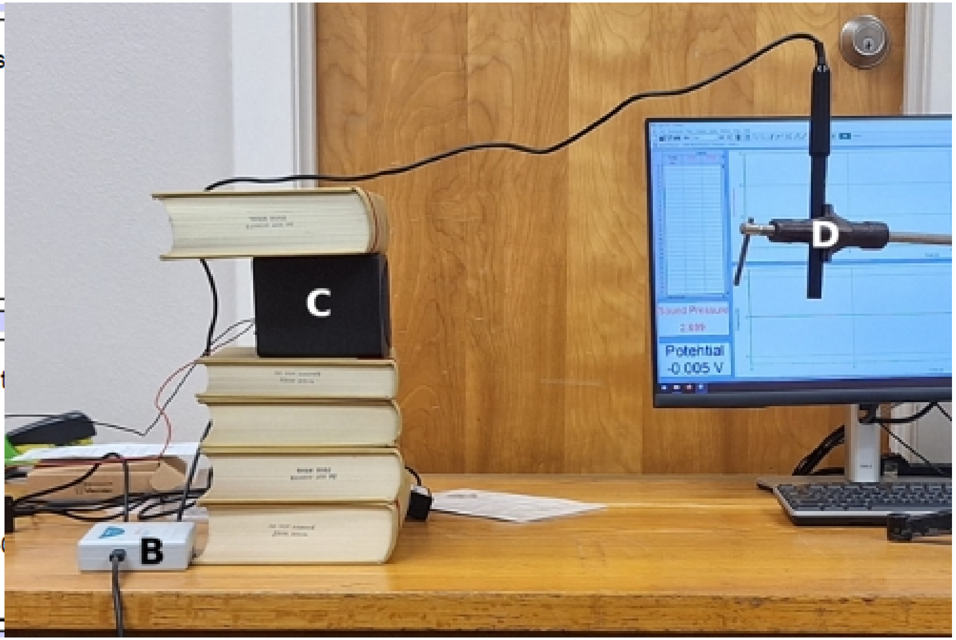
$$\underbrace{p_M(t)}_{\text{microphone}} = \underbrace{p_S(t)}_{\text{speaker}} + \underbrace{p_B(t)}_{\text{bottle}}.$$



Experimental signal

- Easy to measure signal amplitude with and without the bottle
- By monitoring the speaker, we can also measure the phase shift





Normalizing the signal + extracting fit parameters

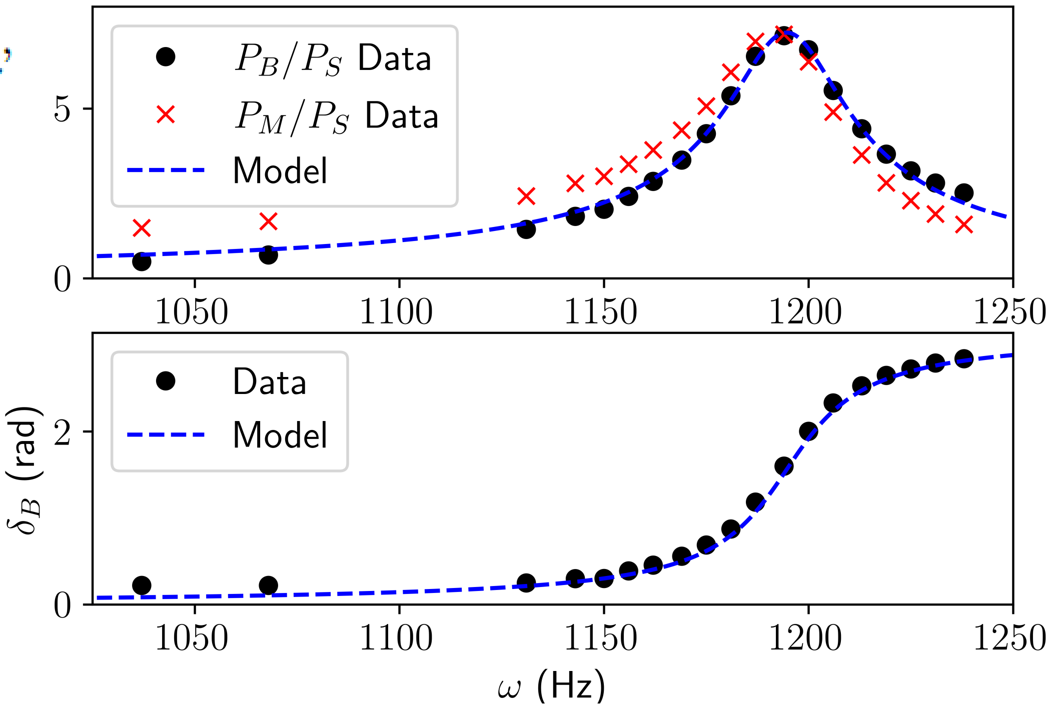
- For pure tones,

$$\underbrace{P_S \cos(\omega t)}_{\text{speaker}} + \underbrace{P_B \cos(\omega t - \delta_B)}_{\text{bottle}} = \underbrace{P_M \cos(\omega t - \delta_M)}_{\text{microphone}},$$

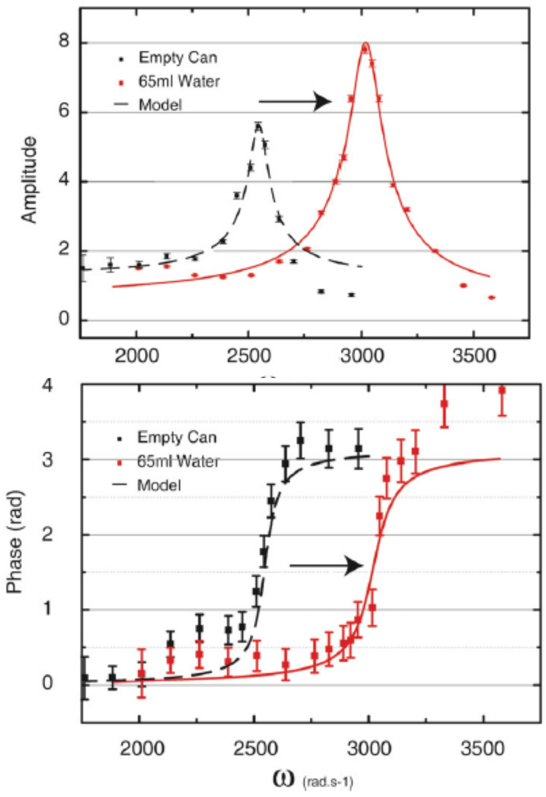
$$P_S e^{i\omega t} + P_B e^{i(\omega t - \delta_B)} = P_M e^{i(\omega t - \delta_M)}$$

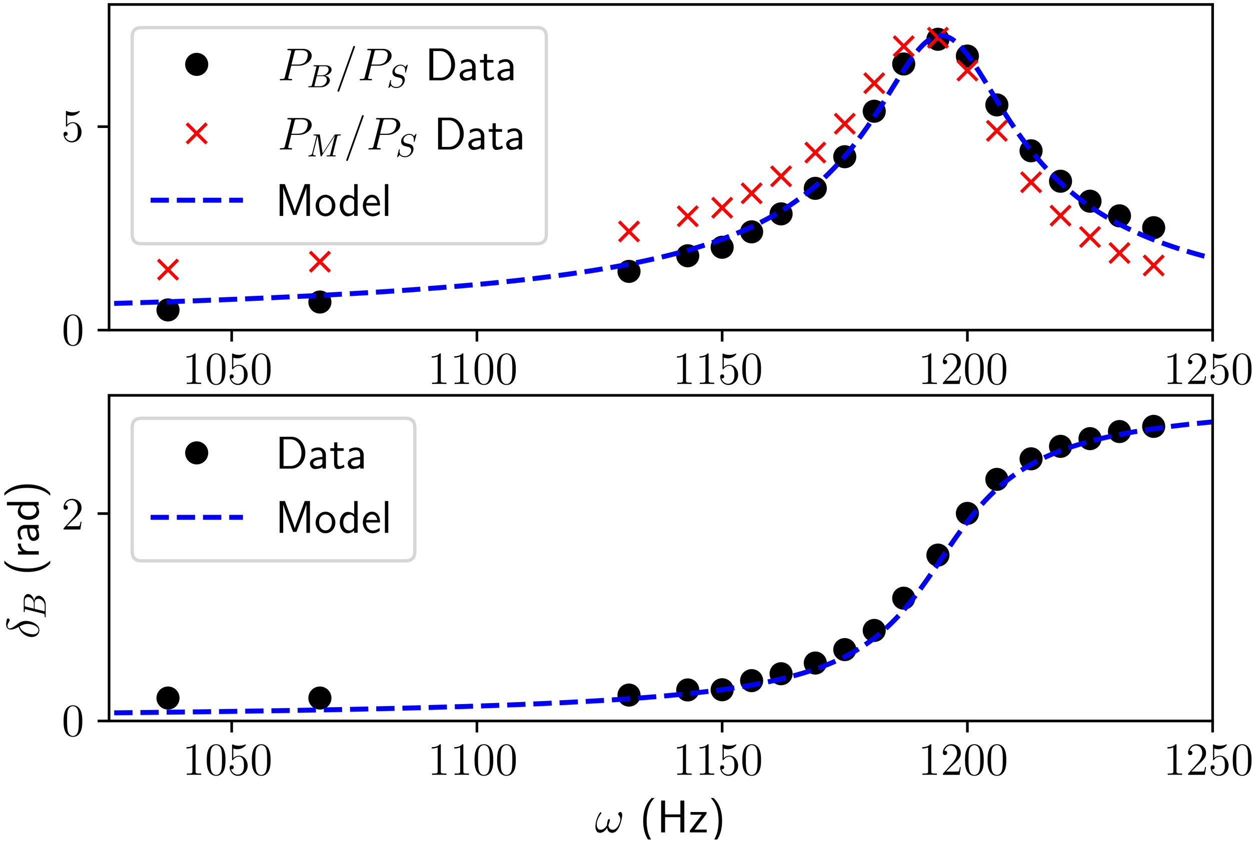
- Normalized amplitudes and phase shifts are fit well by DDO model

$$P_B = \sqrt{P_S^2 + P_M^2 - 2P_S P_M \cos(\delta_M)}$$

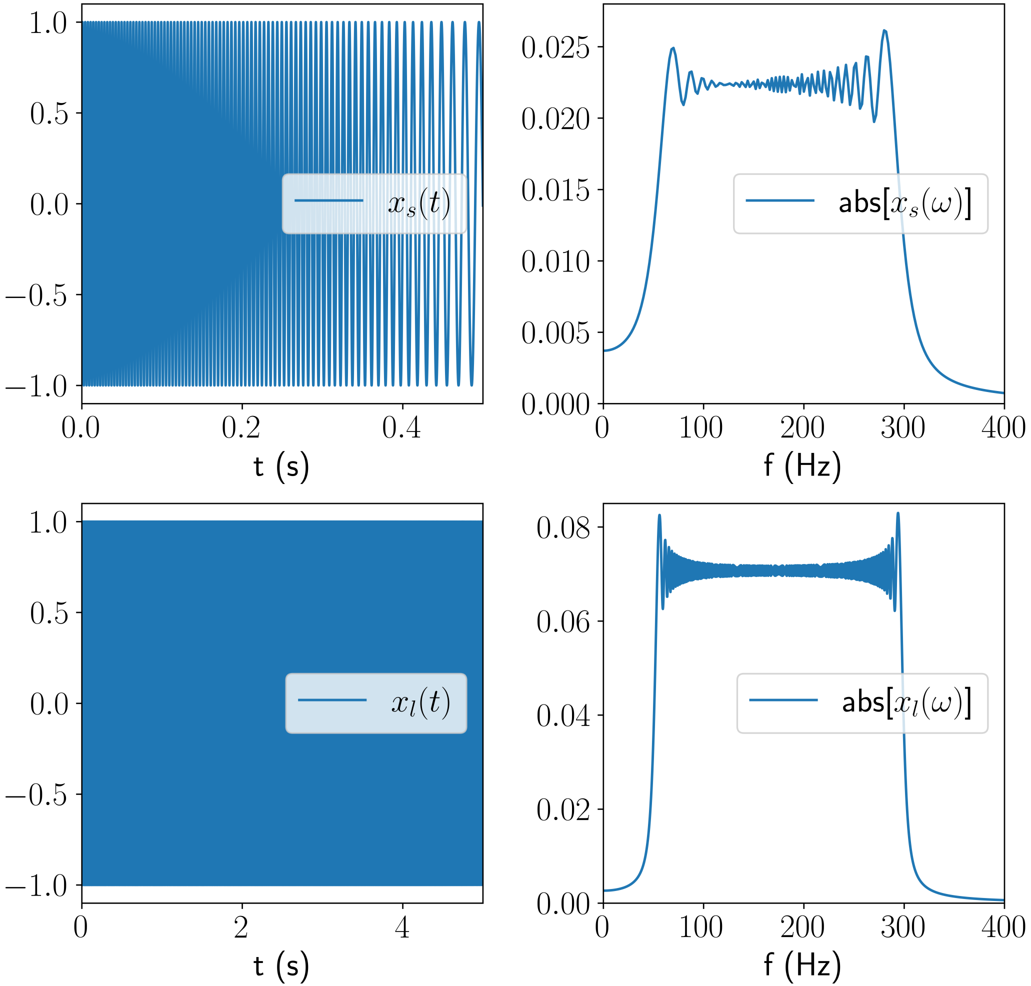


Big improvement over previous fit!





Chirp signals



- Chirp signals are easy to generate using Audacity

$$x(t) = \sin\left[2\pi\left(\frac{c}{2}t^2 + f_0 t\right)\right]$$

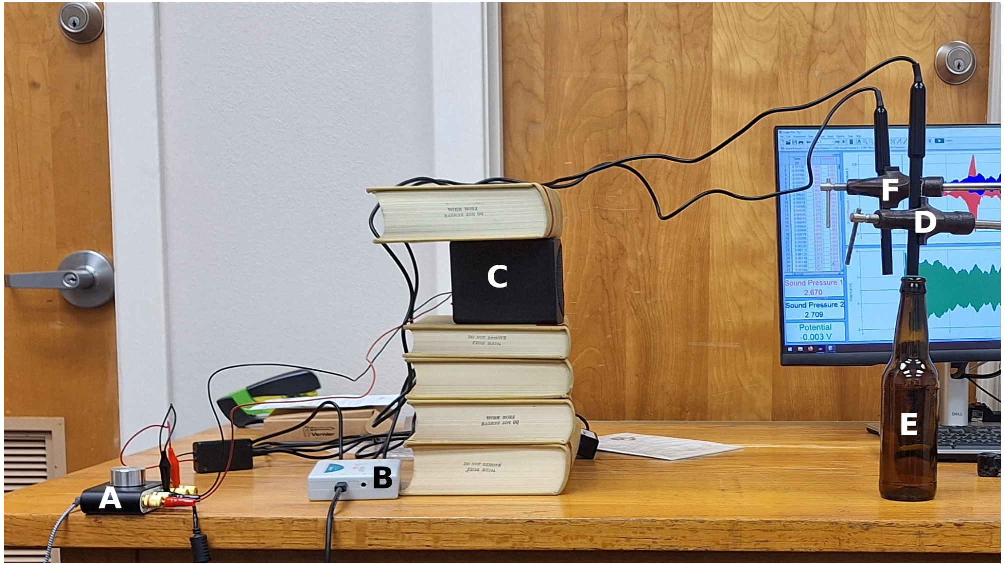
$$c = \frac{f_1 - f_0}{T}.$$

- Chirp spectrum ranges from f_0 to f_1 with a flattish amplitude

Alternative experimental setup

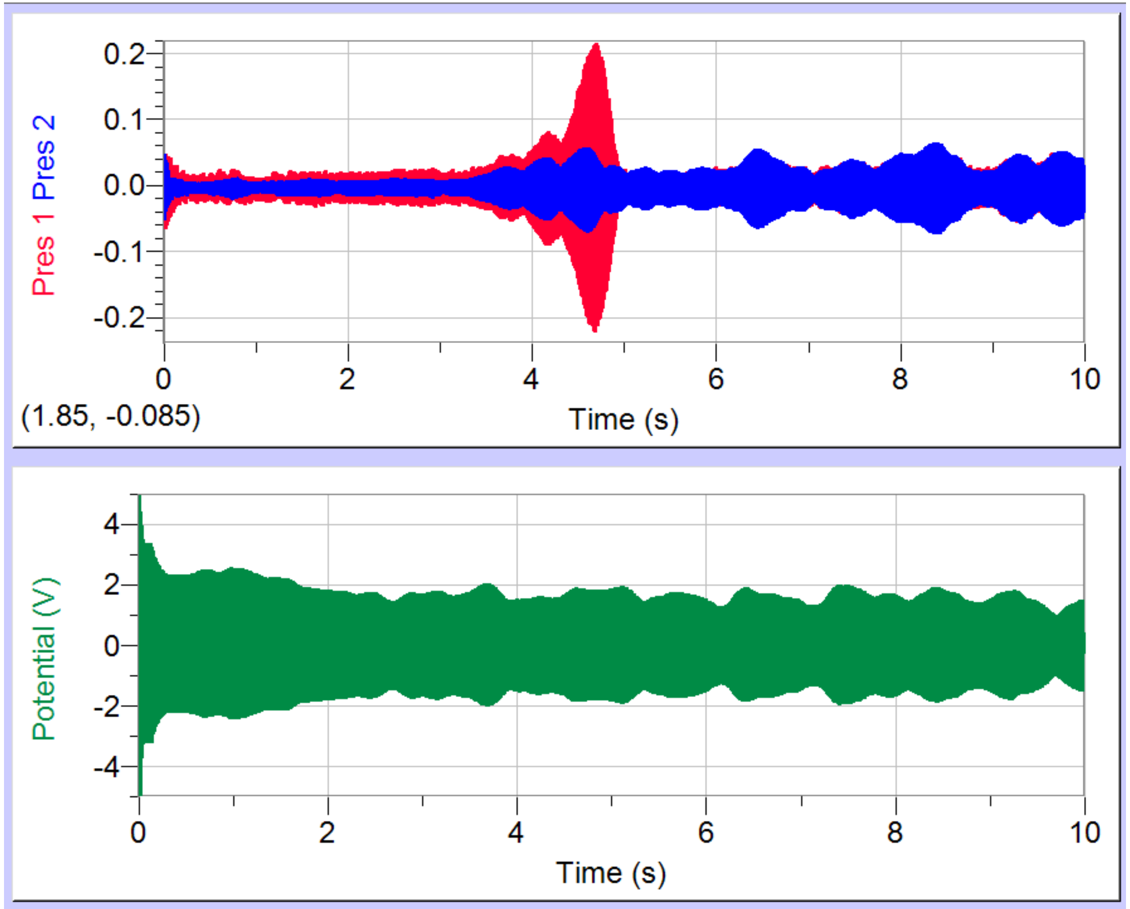
- **Use a chirp signal** to extract DDO parameters efficiently

- An approximation: **both mics** are driven by the **same background signal**, up to an unknown time shift



Using the chirp: Raw signal/response

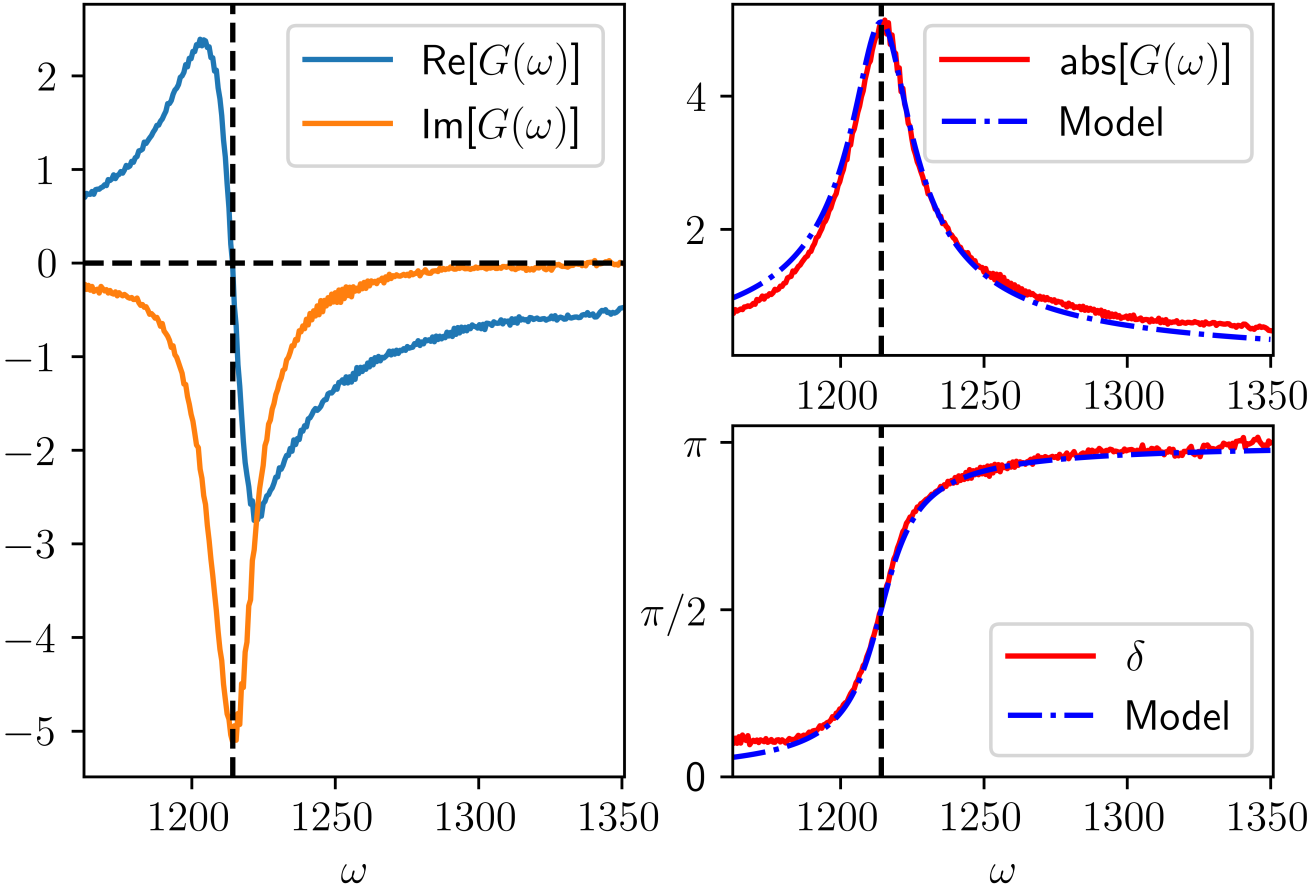
- Blue: signal from the mic without a bottle
- Red: signal from the mic with a bottle
- Green: voltage driving the passive speaker



Extracting Green's function from coherent measurement

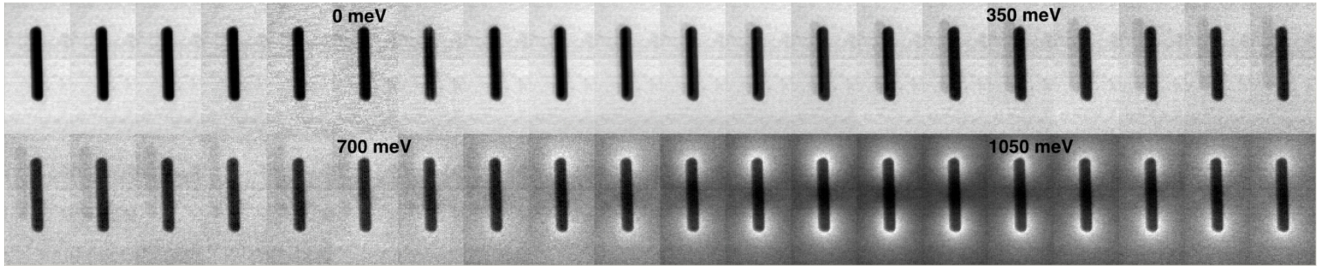
- The details of finding $G(\omega)$ lie in the details of the **Fourier methods**

- Gives us both **phase** and **magnitude**!



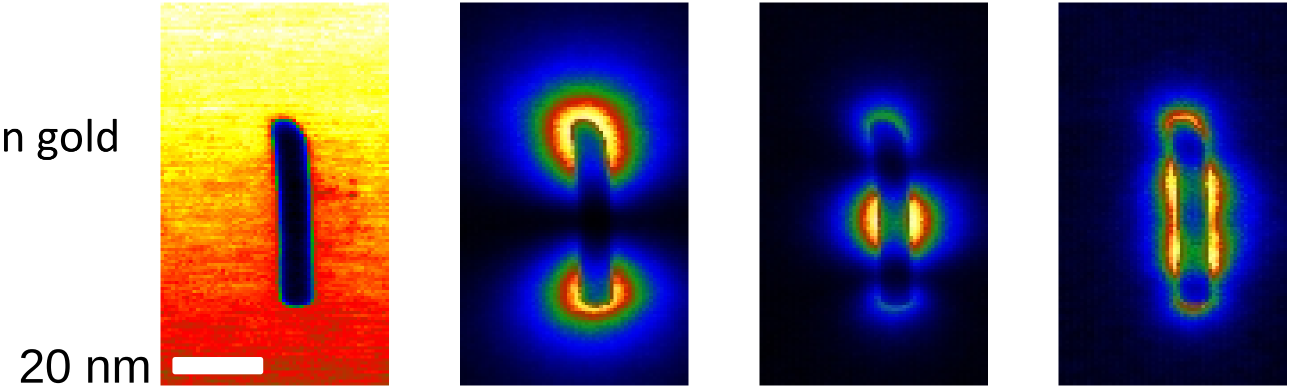
Project 2: Plasmons in nanorods

- How can **resonances** be explored **in mesoscopic systems**?



Spectrum imaging of Au nanorod (60 kV, 100 msec, 5 meV/ch)

Mapping of surface plasmon (SP) modes in gold nano-antannae

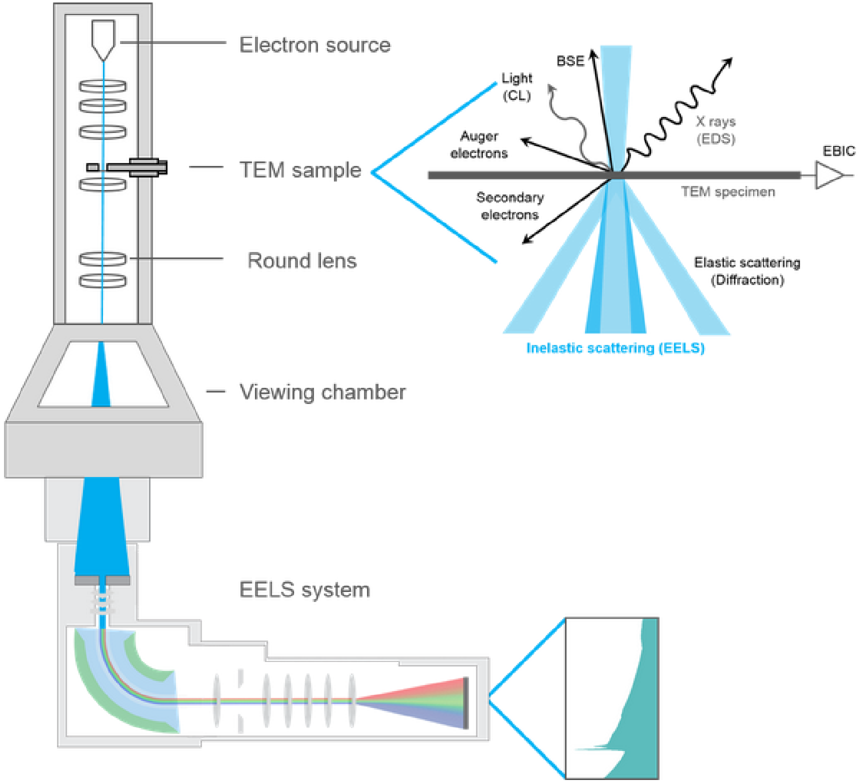


Zero loss | SP mode 1 | SP mode 2 | SP mode 3

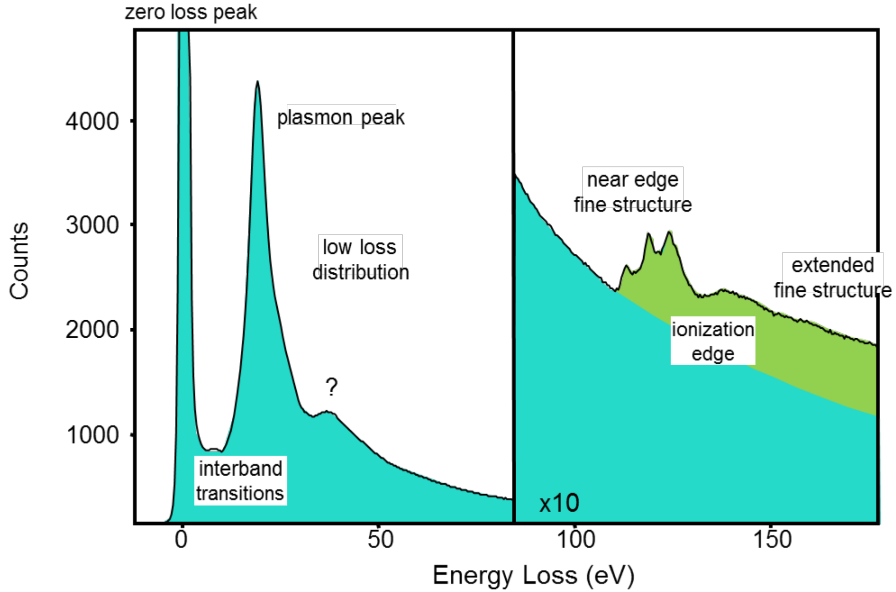
Images are courtesy of Christian Dwyer

Quantum experimental setup: Intro to STEM-EELS

- **STEM = Scanning Transmission Electron Microscope**



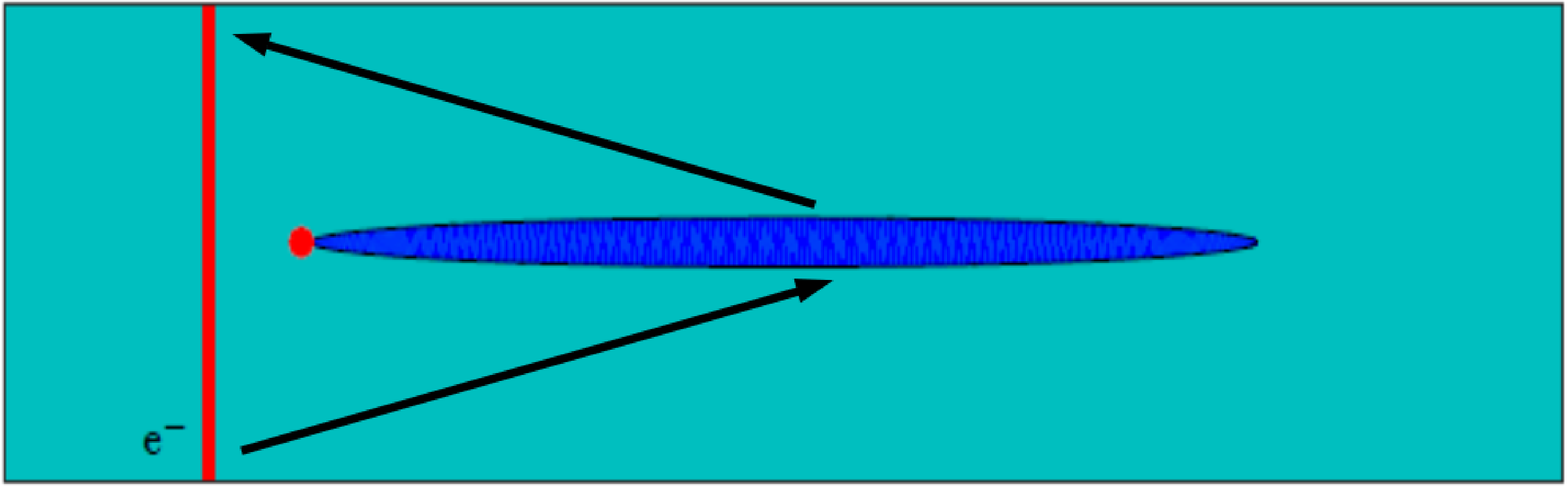
https://eels.info/about/overview



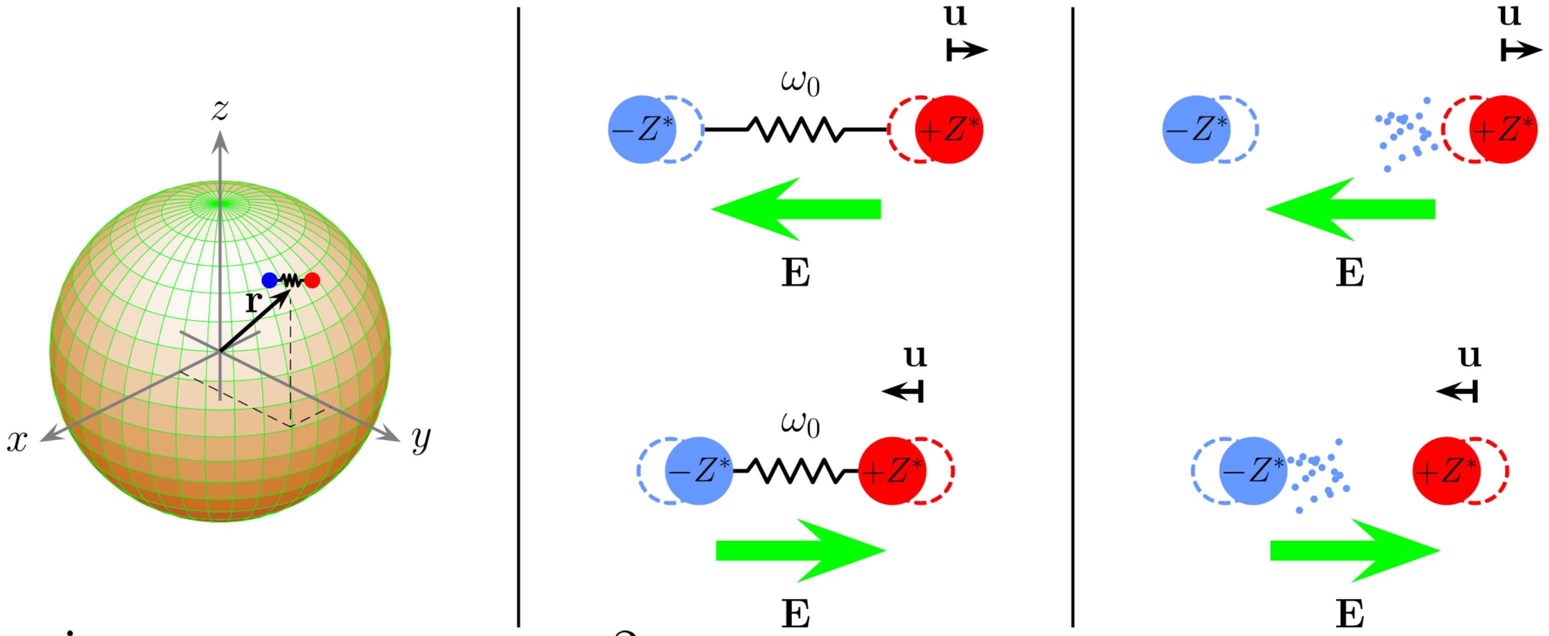
- EELS = Electron Energy-Loss Spectroscopy

Understanding the Physics

- The **nanorod screens the beam's electric field**, and hence contributes to the total electric field
- The nanorod's electric field contribution **does work** on the electron, causing the electron to slow



Born-Huang theory for ionic particles



$$Z_0^* \mathbf{E}^{\text{in}} = \ddot{\mathbf{u}} + 2\eta \dot{\mathbf{u}} + \omega_0^2 \mathbf{u},$$

$$\mathbf{P} = Z_0^* \mathbf{u} + \frac{\epsilon_\infty - 1}{4\pi} \mathbf{E}^{\text{in}}.$$

(Image by Christian Dwyer)

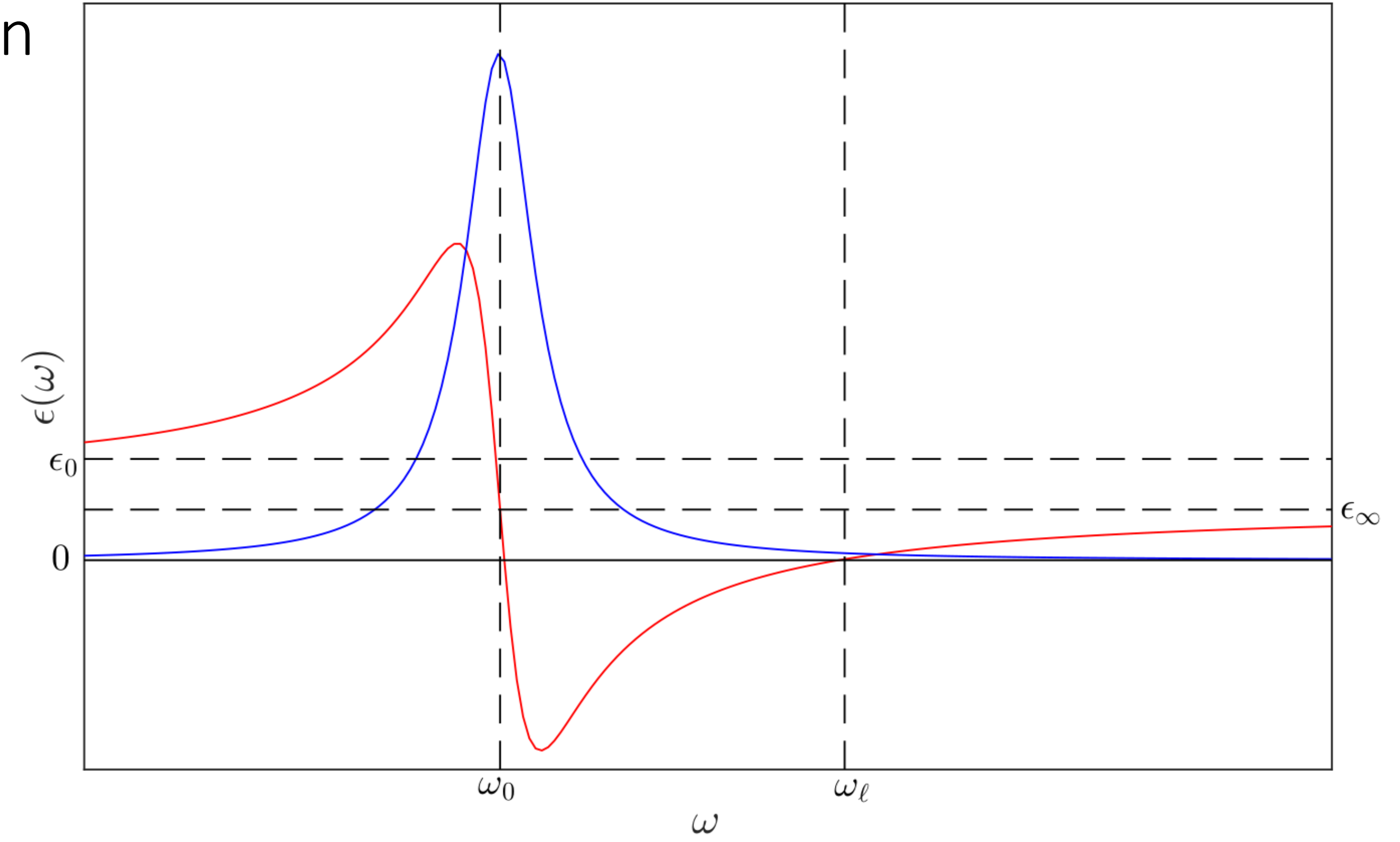
Born-Huang dielectric function

- Dielectric function ϵ(ω) connects input E-fields to output E-fields

$$\nabla \cdot \mathbf{D} = \nabla \cdot \epsilon \mathbf{E} = \nabla \cdot (\mathbf{E} + 4\pi \mathbf{P}) = 0$$

$$\downarrow$$

$$\epsilon(\omega) = \epsilon_\infty \frac{\omega(\omega + 2i\eta) - \omega_\ell^2}{\omega(\omega + 2i\eta) - \omega_0^2}$$



Harmonic functions, harmonic frequencies

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$

- Introduce harmonic functions for a particular geometry

$$\phi_h^{\text{in}} = \phi_h^{\text{out}}\big|_{\text{surface}}$$

Example geometry: Sphere

$$\phi_{\ell m}^{\text{in}} \propto (r/a)^{\ell} P_\ell^m(\cos\vartheta)\exp(im\varphi)$$
$$\phi_{\ell m}^{\text{out}} \propto (a/r)^{\ell+1} P_\ell^m(\cos\vartheta)\exp(im\varphi)$$

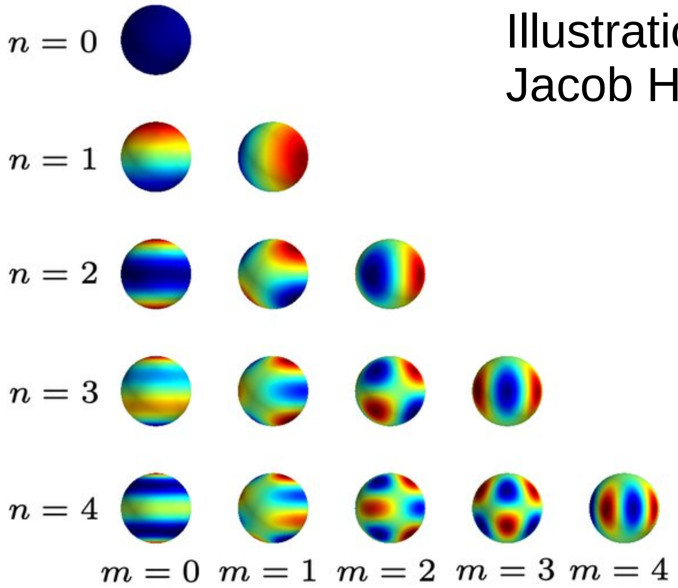
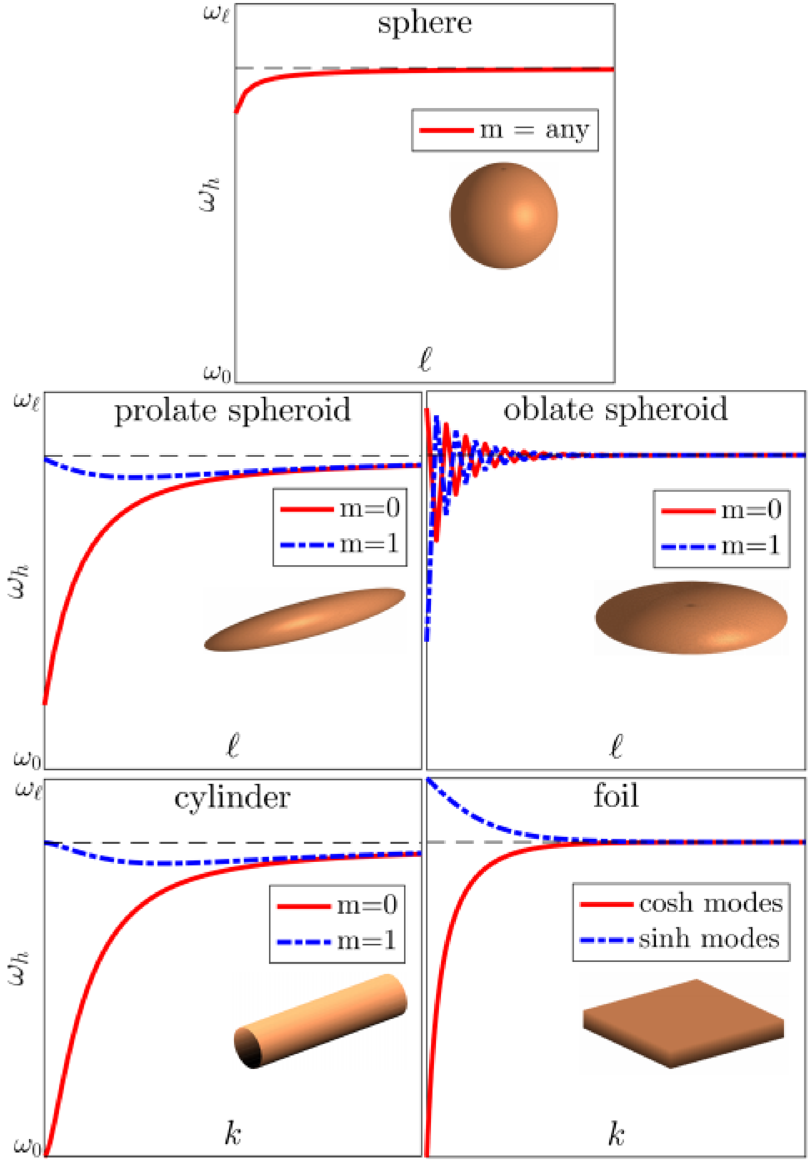


Illustration by Jacob Hollebon

Harmonic Frequencies

- To find the harmonic frequencies…
  - 1. Scale harmonic functions
  - 2. Find mode dielectric values
  - 3. Find mode frequencies



Harmonic Frequencies

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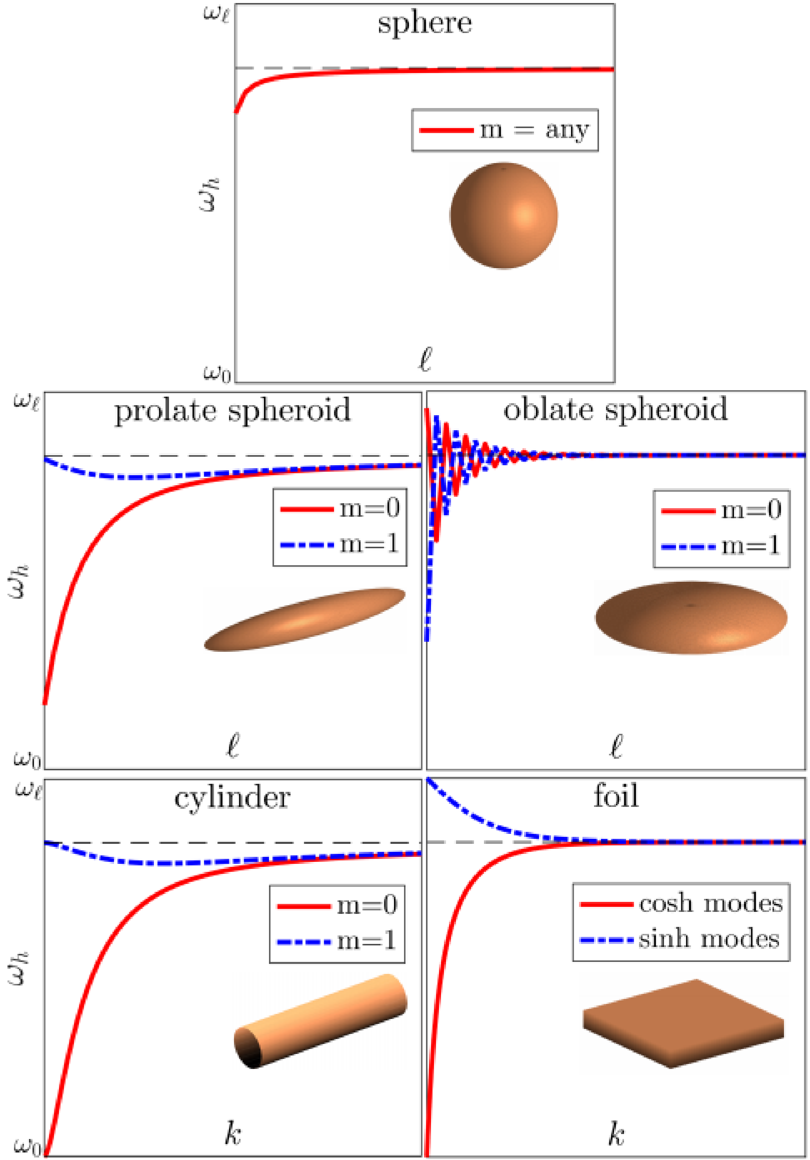
  - 3. Find mode frequencies



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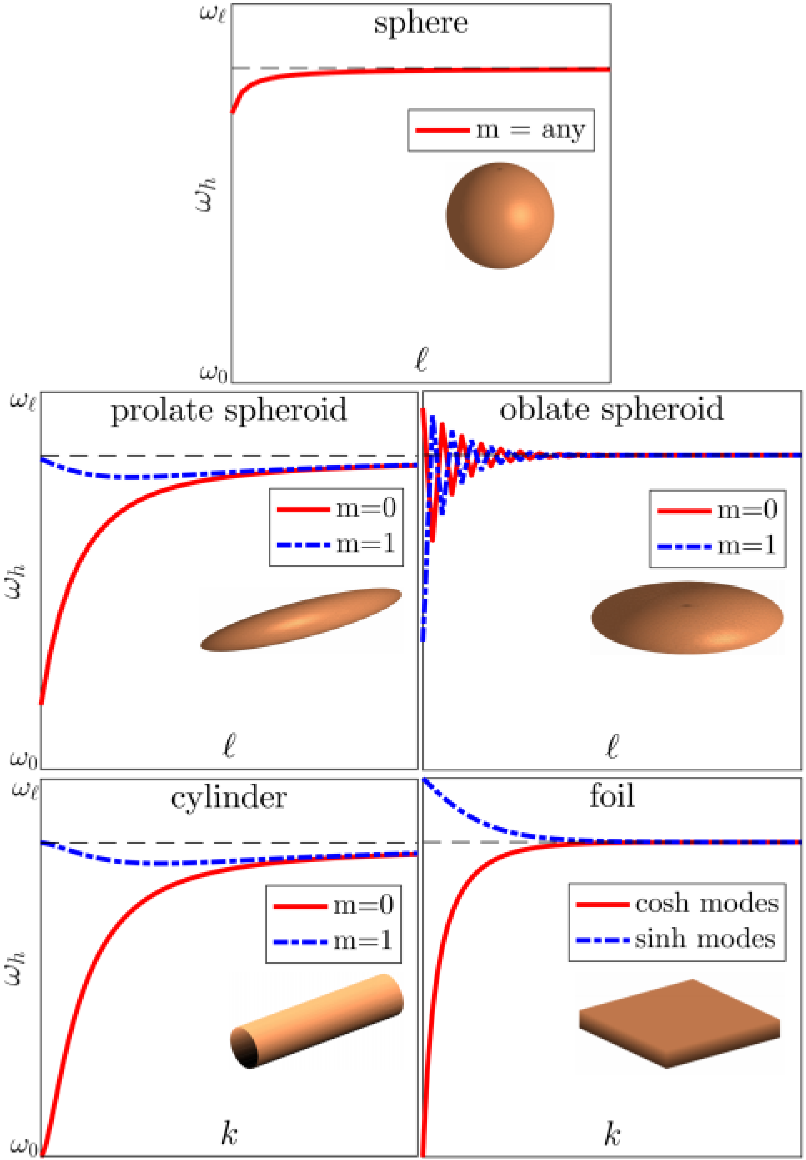
  - **2. Find mode dielectric values**

$$\epsilon_h = \left.\frac{\partial_\perp \phi_h^{\text{out}}}{\partial_\perp \phi_h^{\text{in}}}\right|_{\text{surface}} \Rightarrow \epsilon_{\ell m} = -\frac{\ell+1}{\ell}$$

  - 3. Find mode frequencies



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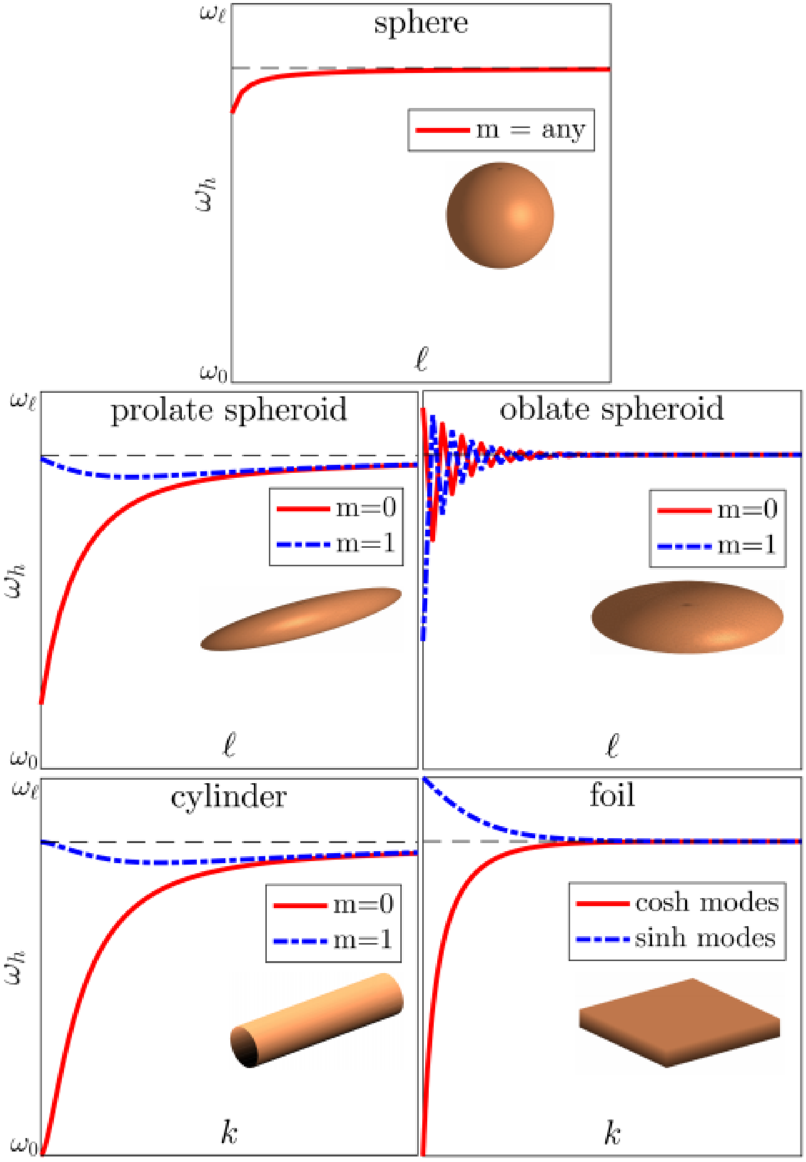
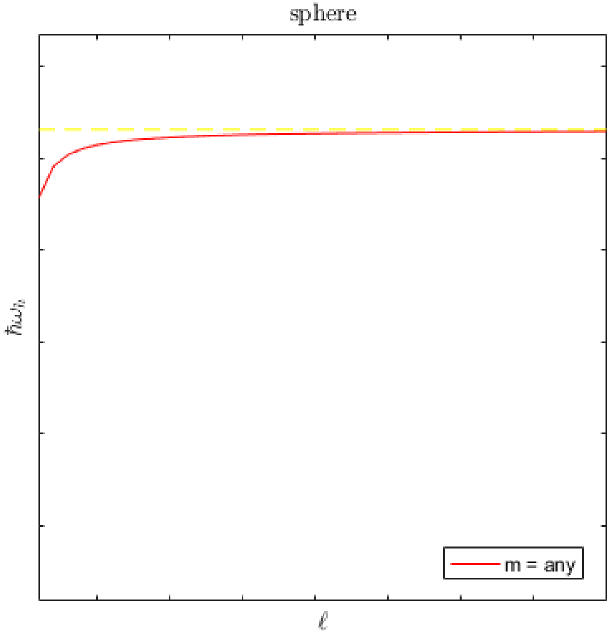
  - 3. Find **mode frequencies**

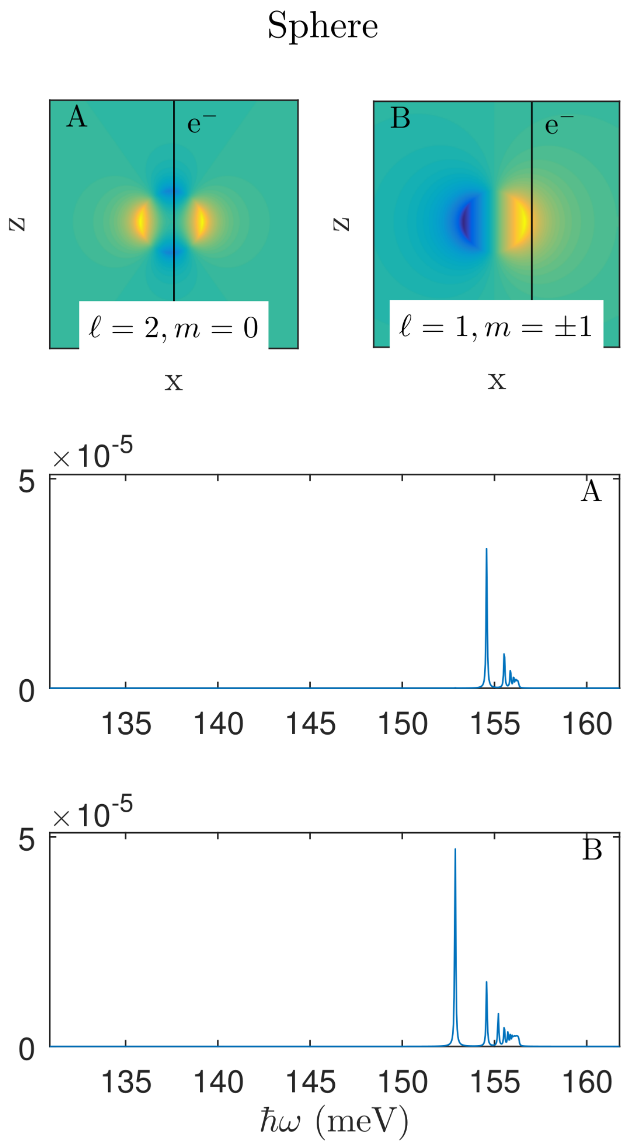
$$\epsilon(\omega_h) = \epsilon_h \Rightarrow \omega_{\ell m}^2 = \frac{\ell\epsilon_\infty\omega_\ell^2 + (\ell+1)\omega_0^2}{\ell\epsilon_\infty + (\ell+1)}$$



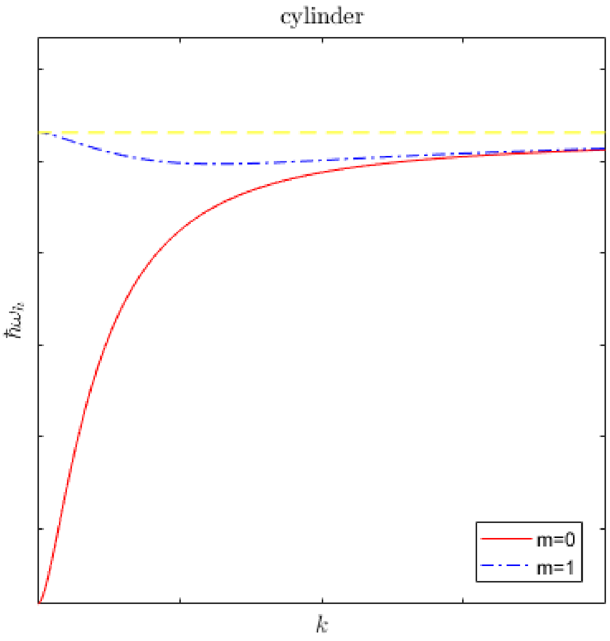
$$\phi_{\ell m}^{\text{in}} \propto (r/a)^{\ell} P_{\ell}^{m}(\cos\vartheta)\exp(im\varphi)$$

$$\phi_{\ell m}^{\text{out}} \propto (a/r)^{\ell+1} P_{\ell}^{m}(\cos\vartheta)\exp(im\varphi)$$

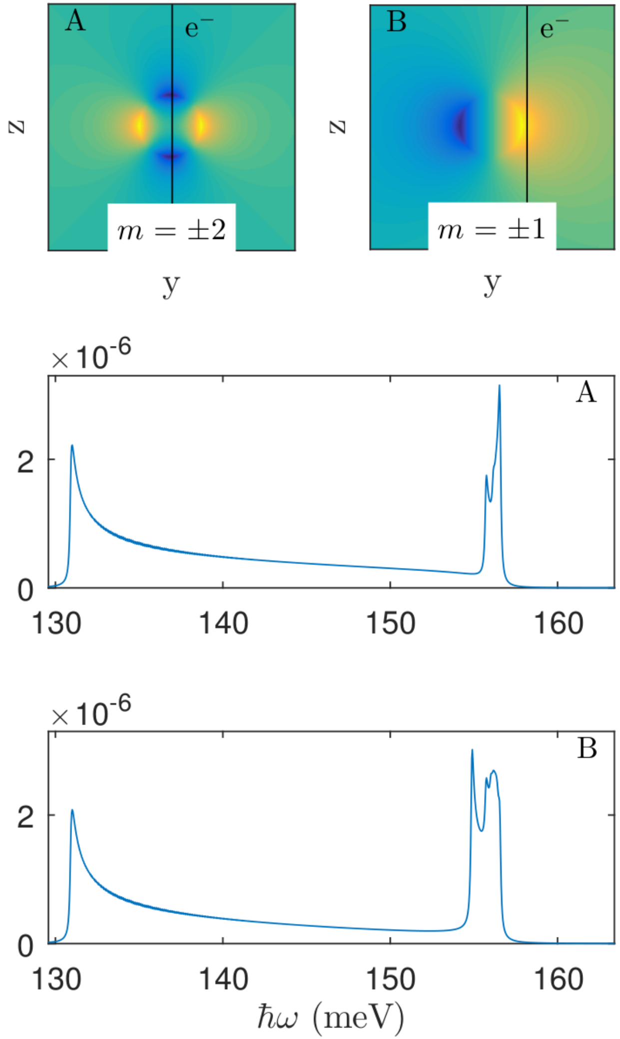




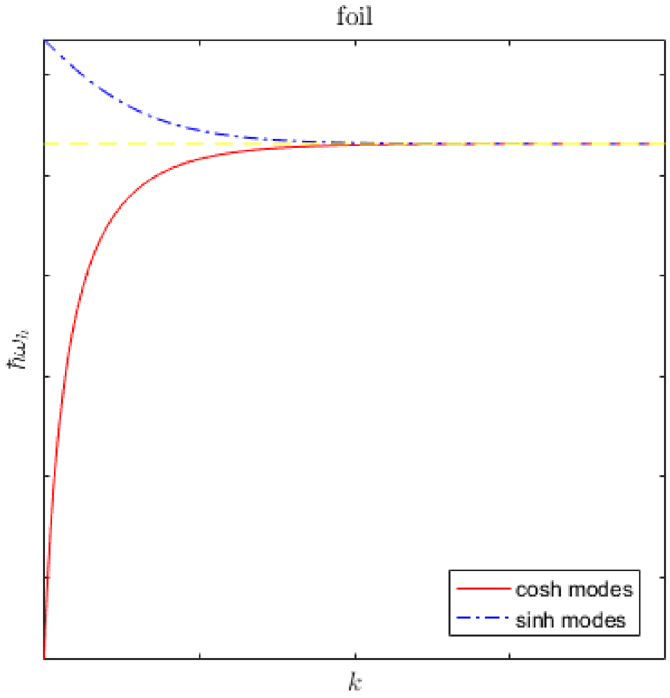
$$\phi_{km}^{\text{in}} \propto \exp(ikx + im\varphi) I_{|m|}(|k|\rho)$$
$$\phi_{km}^{\text{out}} \propto \exp(ikx + im\varphi) K_{|m|}(|k|\rho)$$



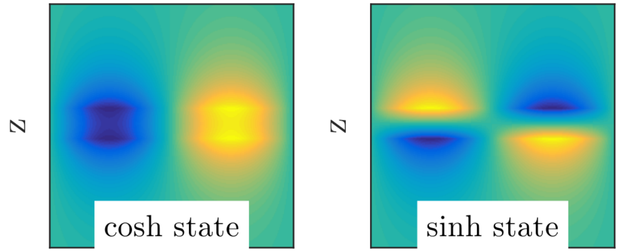
Cylinder

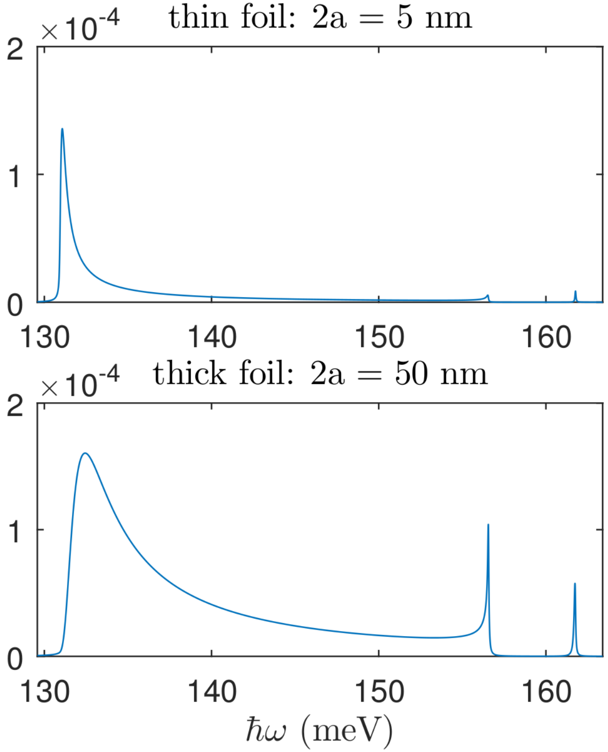


$$\phi^{\text{in}}_{\mathbf{k}} \propto \exp(i\mathbf{k}\cdot\mathbf{x}) \begin{cases} \cosh(|\mathbf{k}|z) \\ \sinh(|\mathbf{k}|z) \end{cases}$$

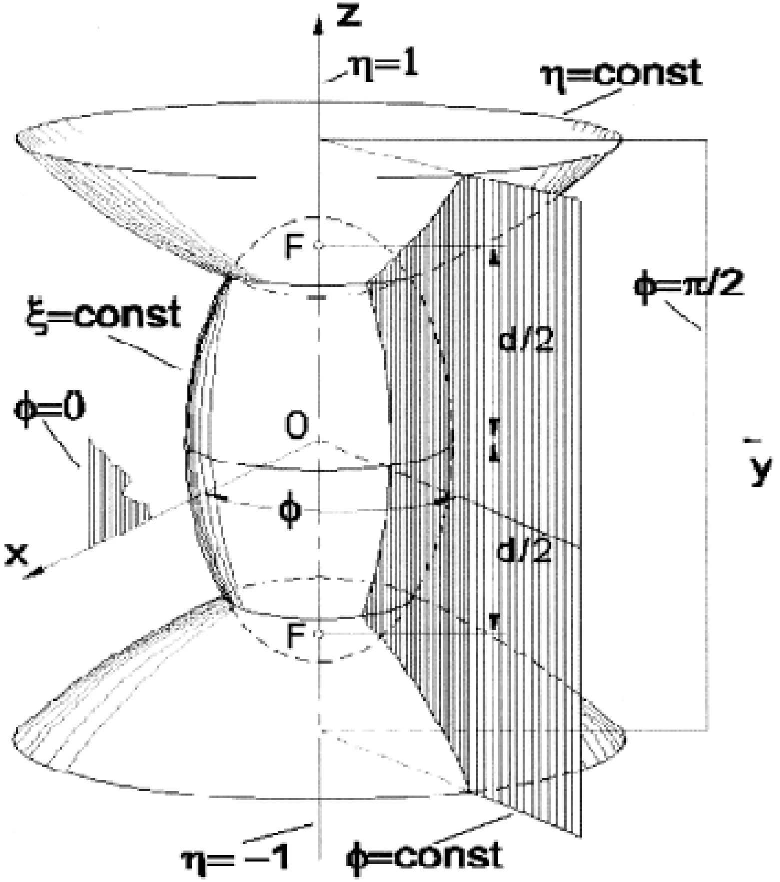


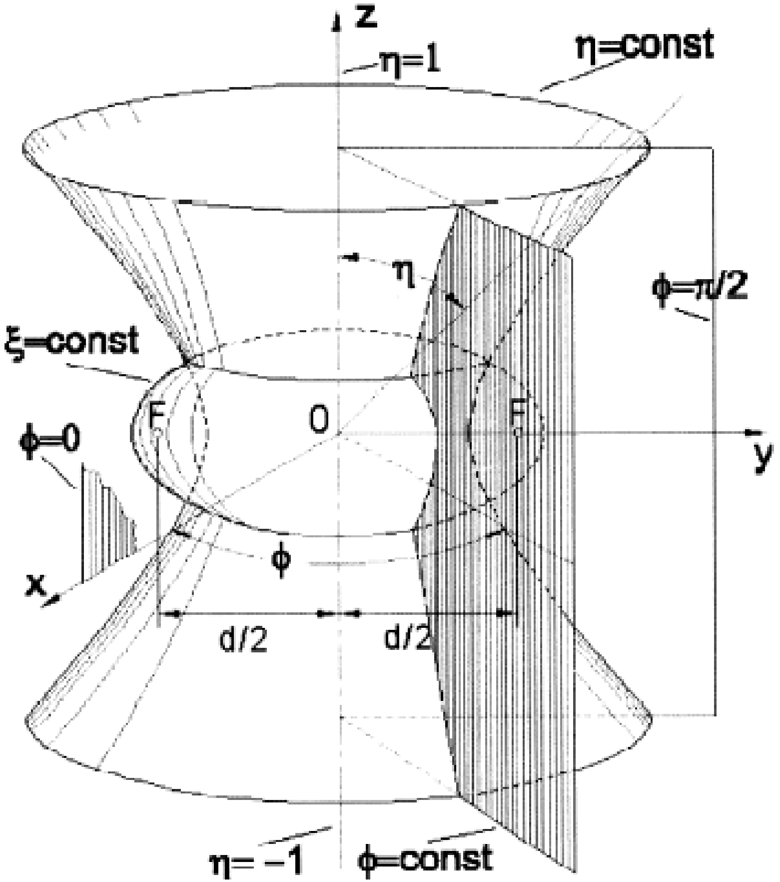
Foil





prolate spheroidal coordinates oblate spheroidal coordinates

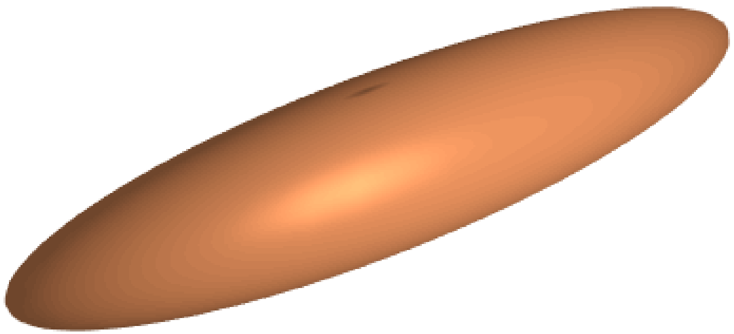




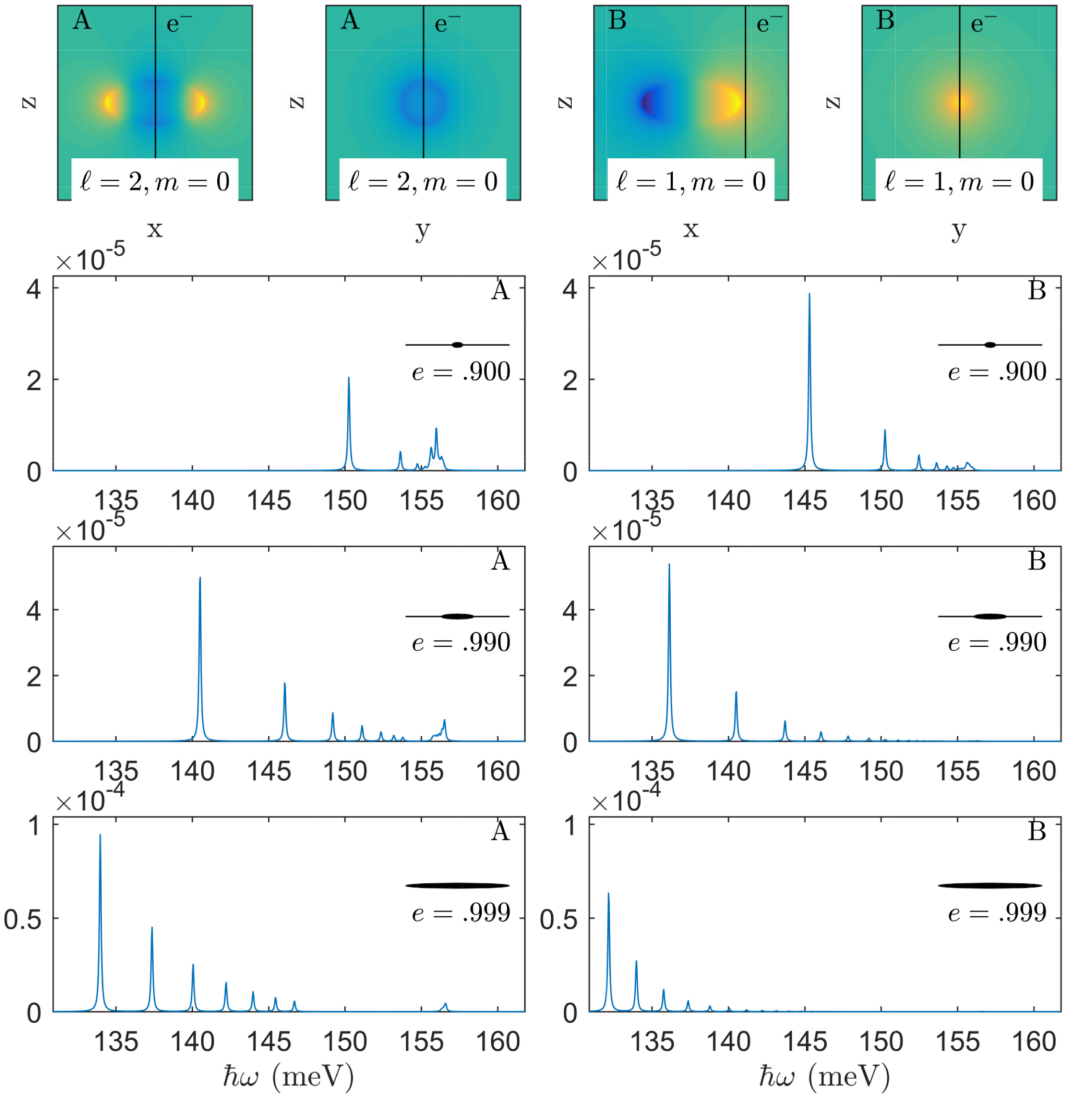
L.-W. Li, X.-K. Kang, and M.-S. Leong, *Spheroidal Wave Functions in Electromagnetic Theory*, John Wiley & Sons, 2002.

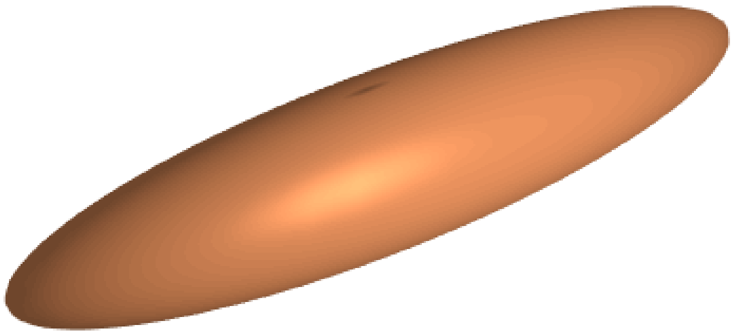
Prolate Spheroids

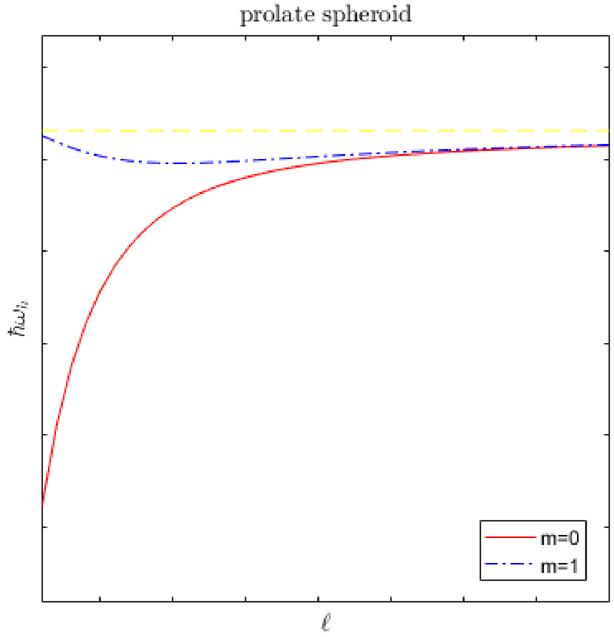
$$x = c\cosh(\varrho)\cos(\vartheta) \qquad 0 \le \varrho < \infty$$
$$y = c\sinh(\varrho)\sin(\vartheta)\cos(\varphi) \qquad 0 \le \vartheta \le \pi$$
$$z = c\sinh(\varrho)\sin(\vartheta)\sin(\varphi) \qquad 0 \le \varphi \le 2\pi$$



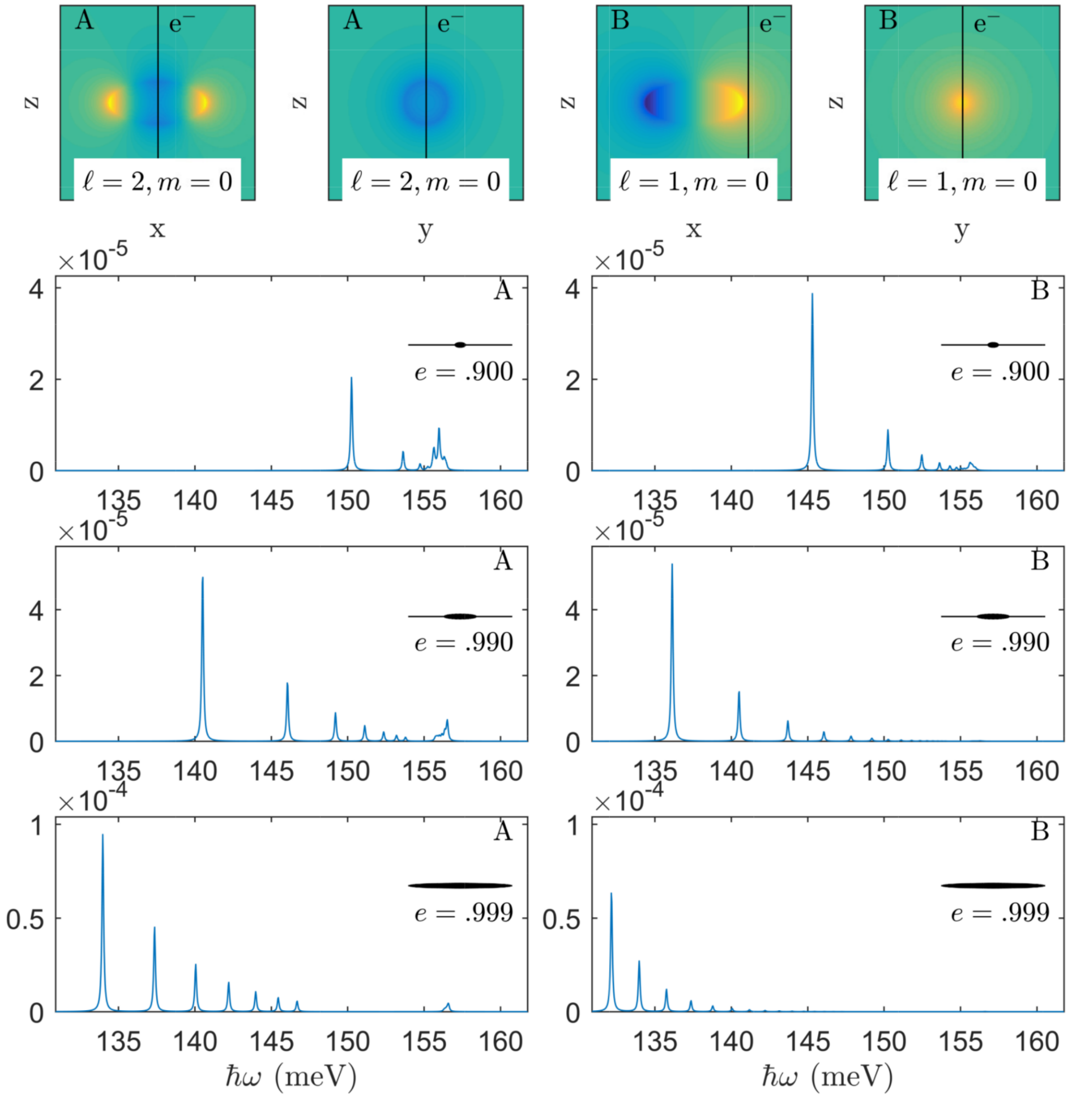
$$\phi_{\ell m}^{\text{in}} \propto P_\ell^m(\cosh\varrho)P_\ell^m(\cos\vartheta)\exp(im\varphi)$$
$$\phi_{\ell m}^{\text{out}} \propto Q_\ell^m(\cosh\varrho)P_\ell^m(\cos\vartheta)\exp(im\varphi)$$



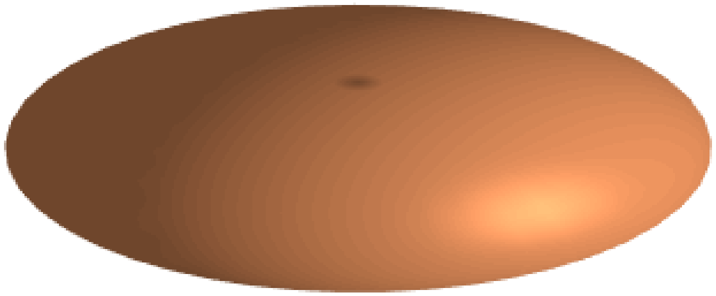




Prolate Spheroids

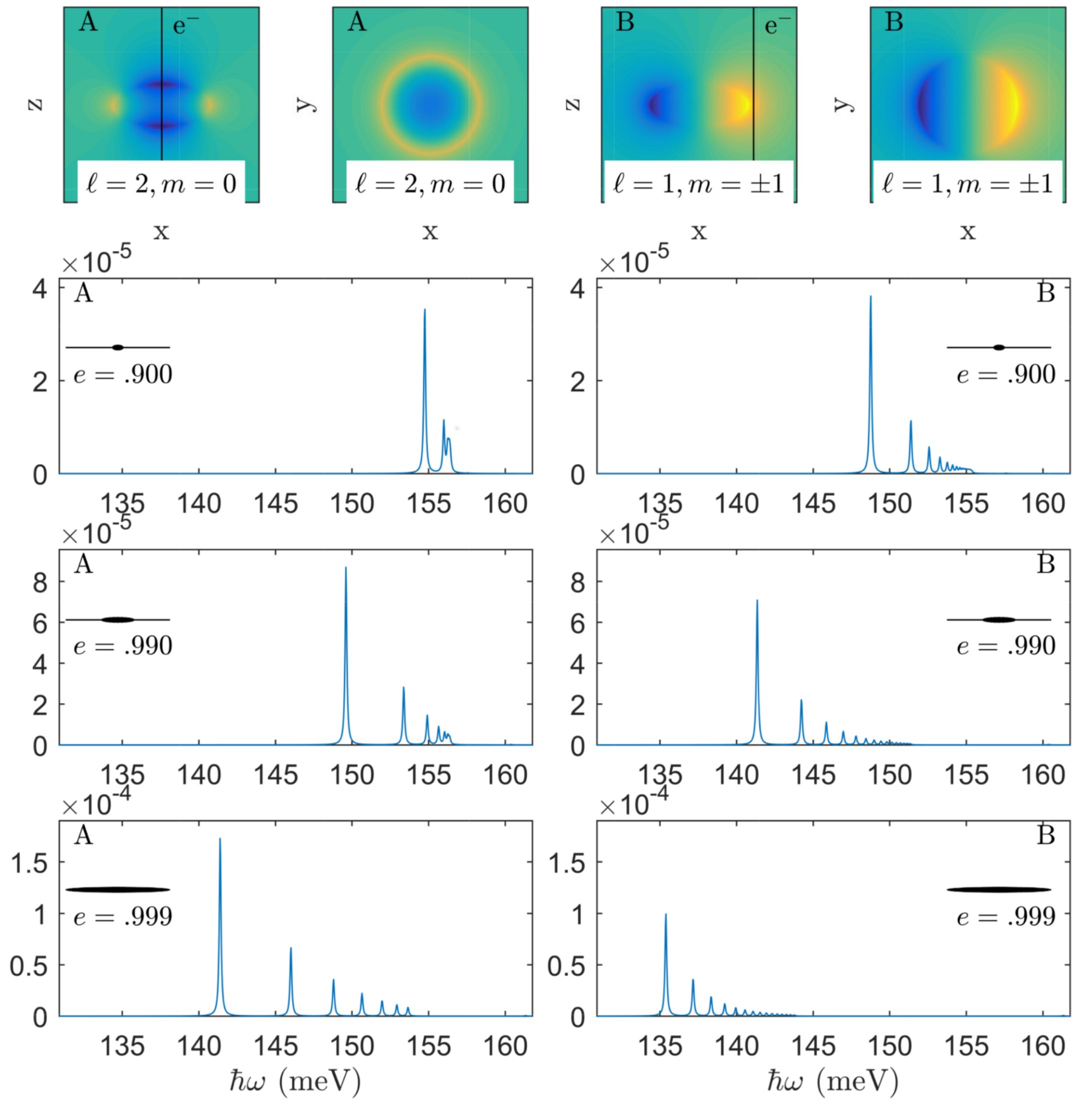


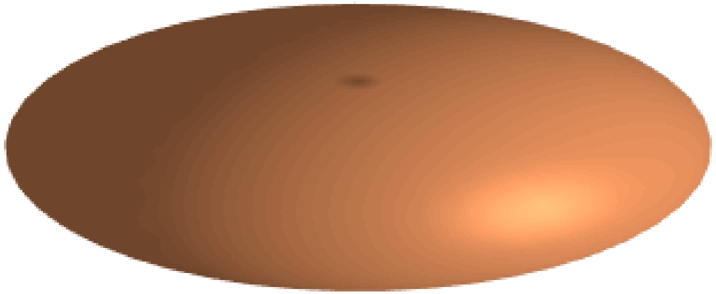
$$x = c\cosh(\varrho)\cos(\vartheta)\cos(\varphi) \qquad 0 \le \varrho < \infty$$
$$y = c\cosh(\varrho)\cos(\vartheta)\sin(\varphi) \qquad -\pi/2 \le \vartheta \le \pi/2$$
$$z = c\sinh(\varrho)\sin(\vartheta) \qquad 0 \le \varphi \le 2\pi$$

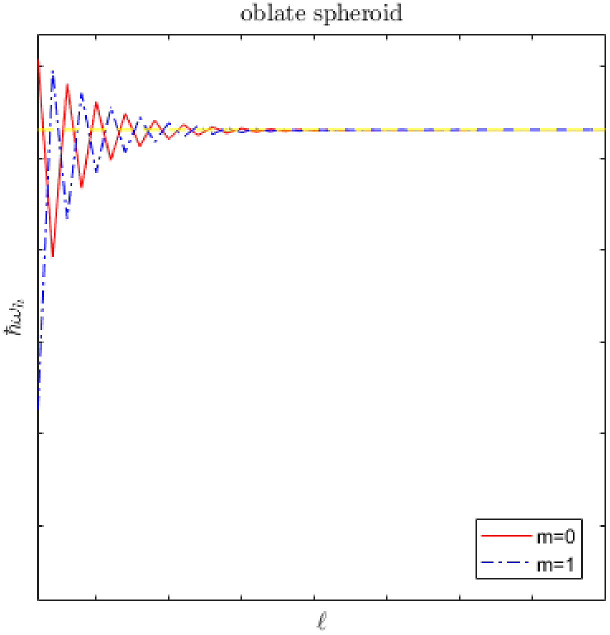


$$\phi^{\text{in}}_{\ell m} \propto P^m_\ell(i\sinh\varrho)P^m_\ell(\sin\vartheta)\exp(im\varphi)$$
$$\phi^{\text{out}}_{\ell m} \propto Q^m_\ell(i\sinh\varrho)P^m_\ell(\sin\vartheta)\exp(im\varphi)$$

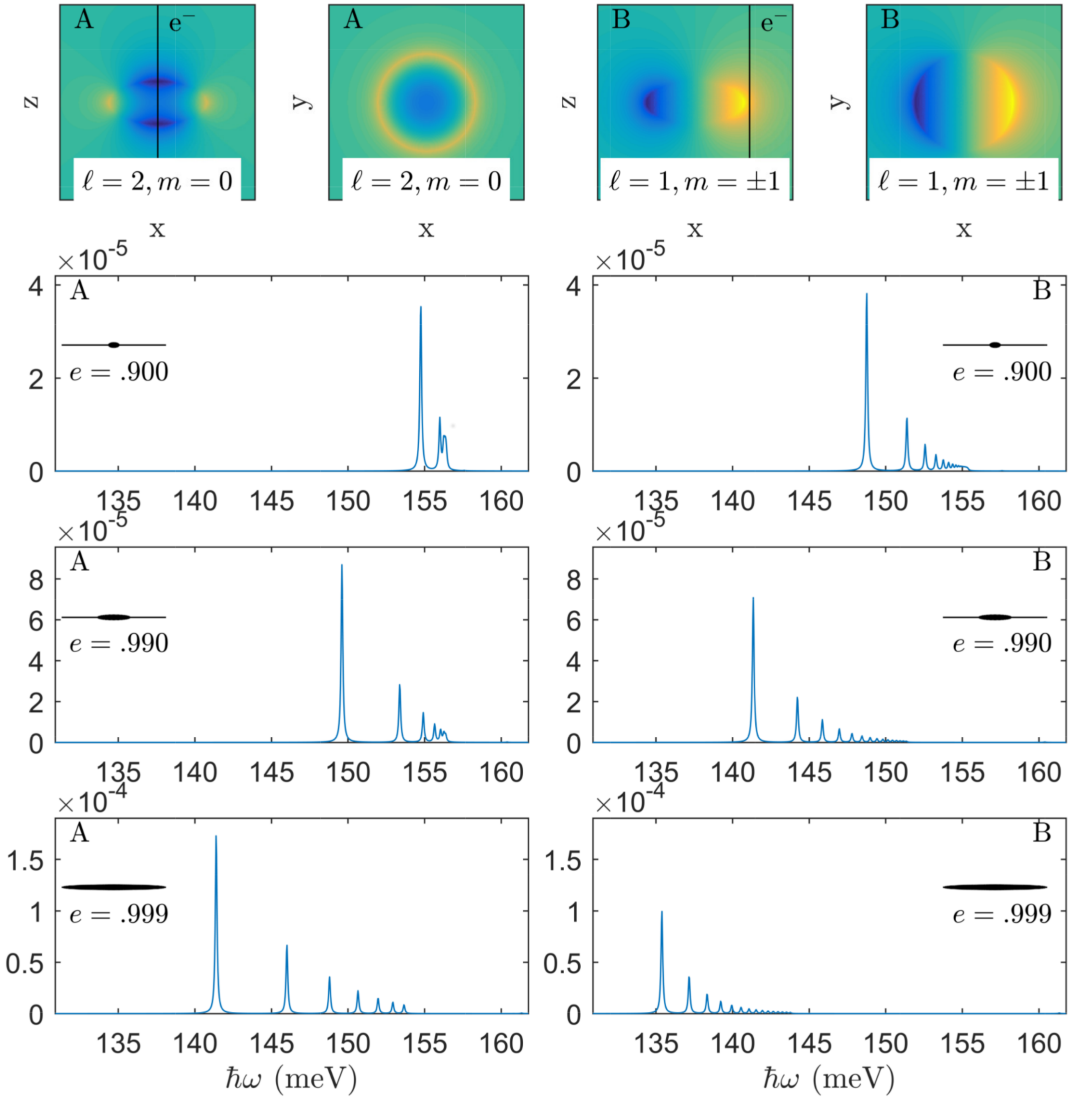
Oblate Spheroids

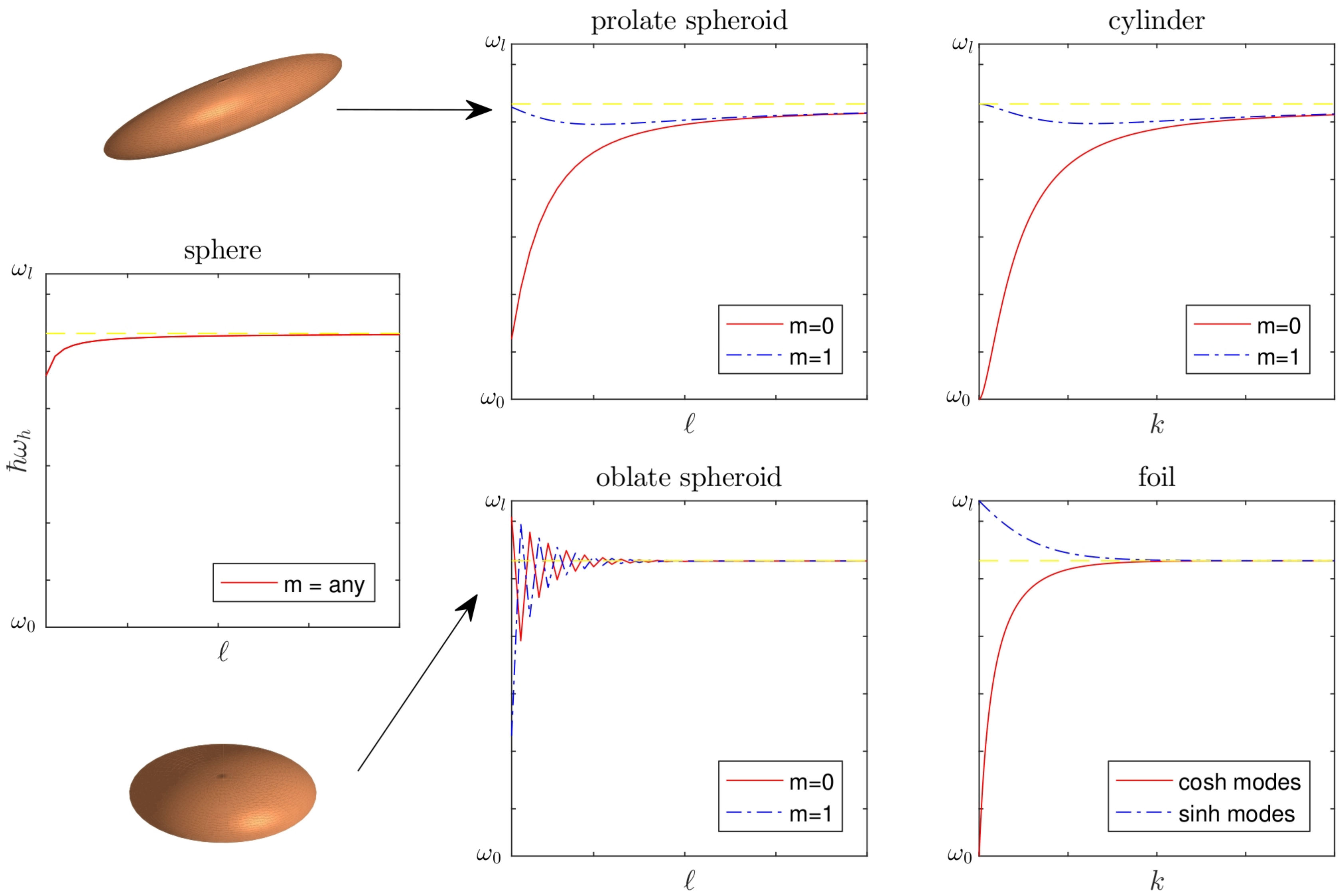






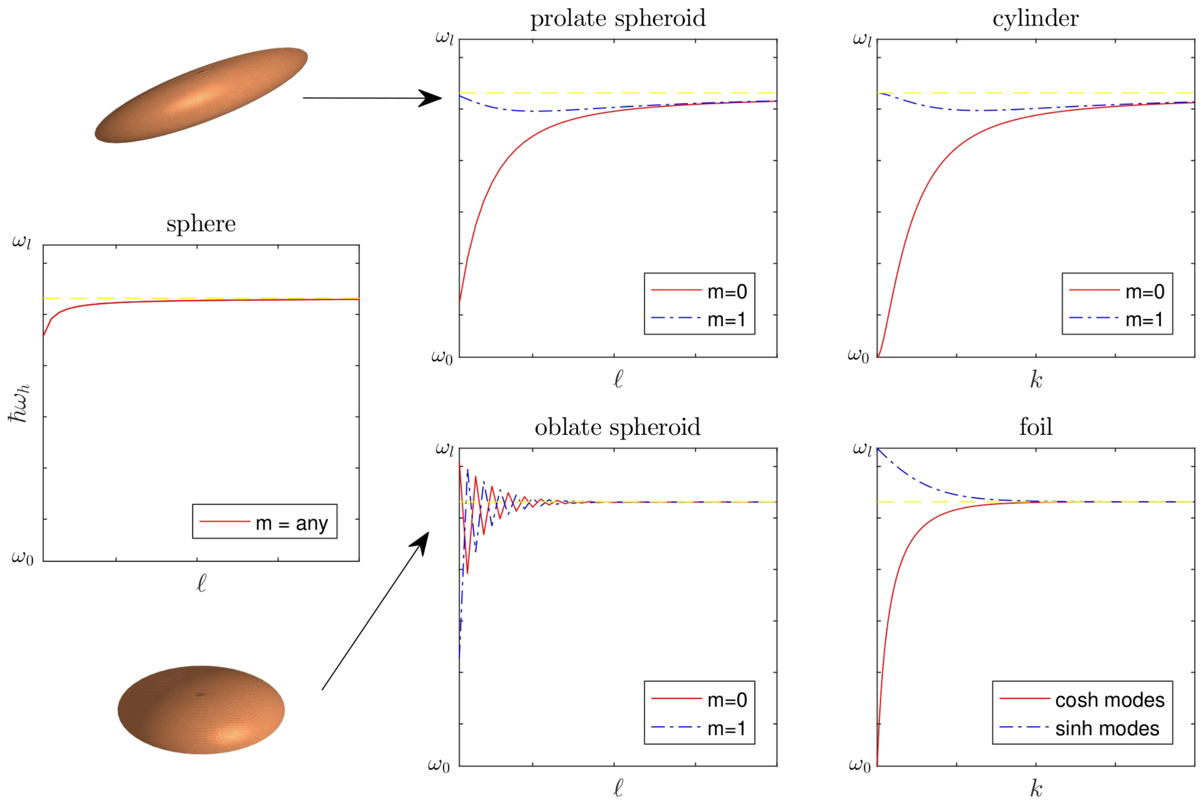
Oblate Spheroids



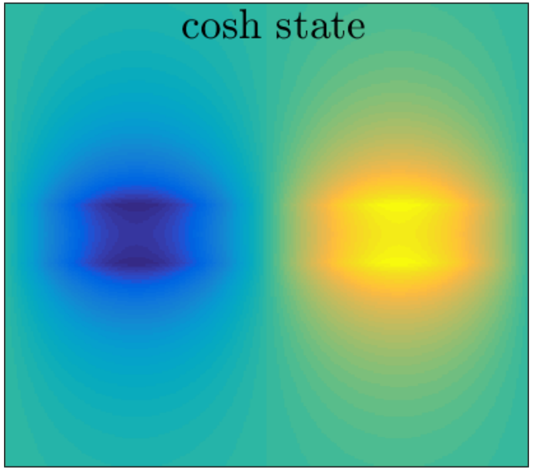


prolate spheroid
cylinder
sphere
oblate spheroid
foil
ω_l
ω_0
$\hbar\omega_h$
ℓ
k
m=0
m=1
m = any
cosh modes
sinh modes

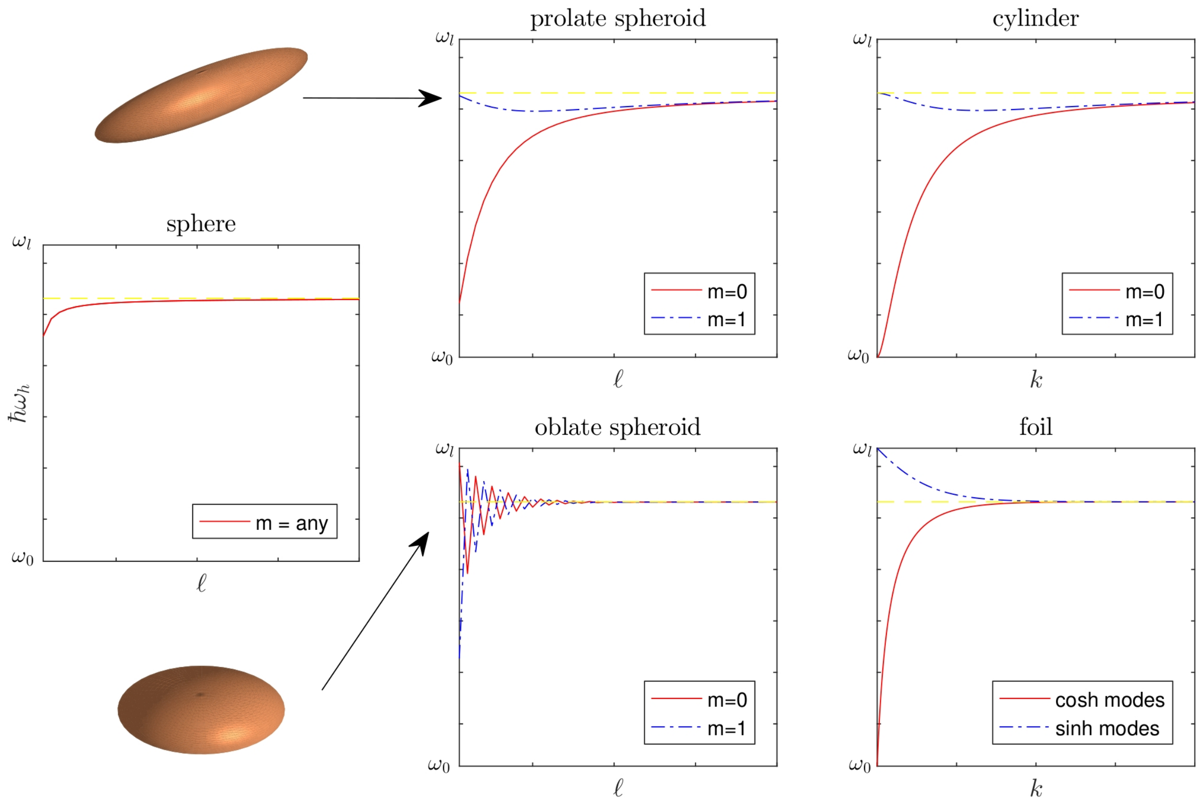
What are the surface states like?



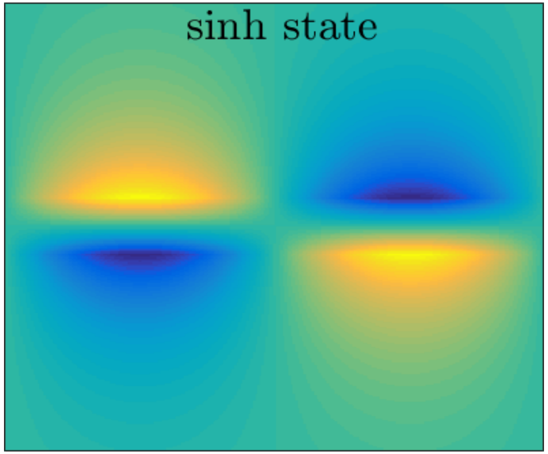
- **Cosh-like states**
  - Low energy
  - Strongly scattering
  - Low density-of-states



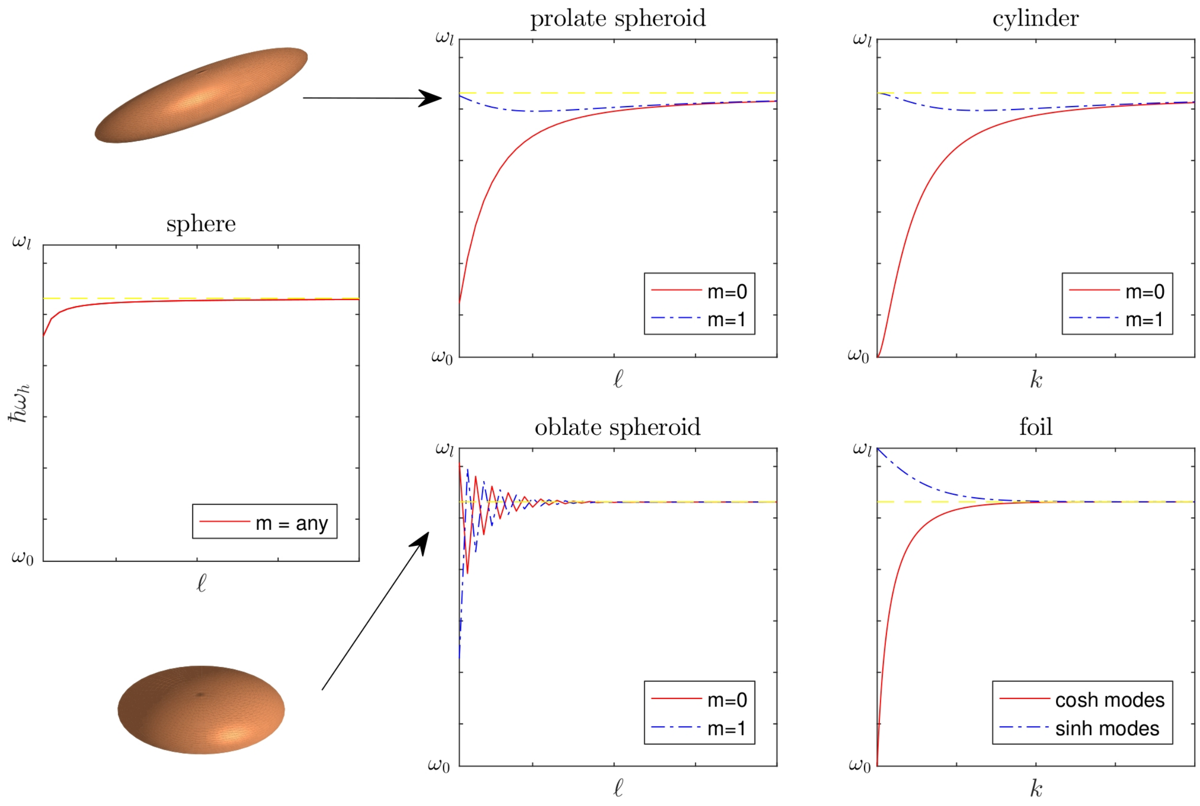
What are the surface states like?



- **Sinh-like states**
  - High energy
  - Weakly scattering
  - Low density-of-states



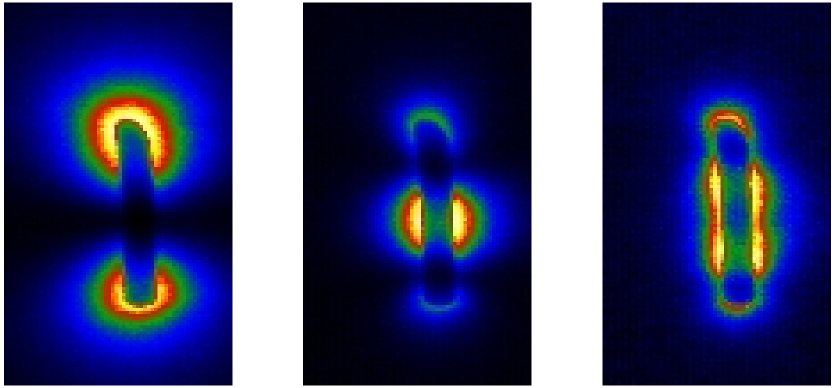
What are the surface states like?



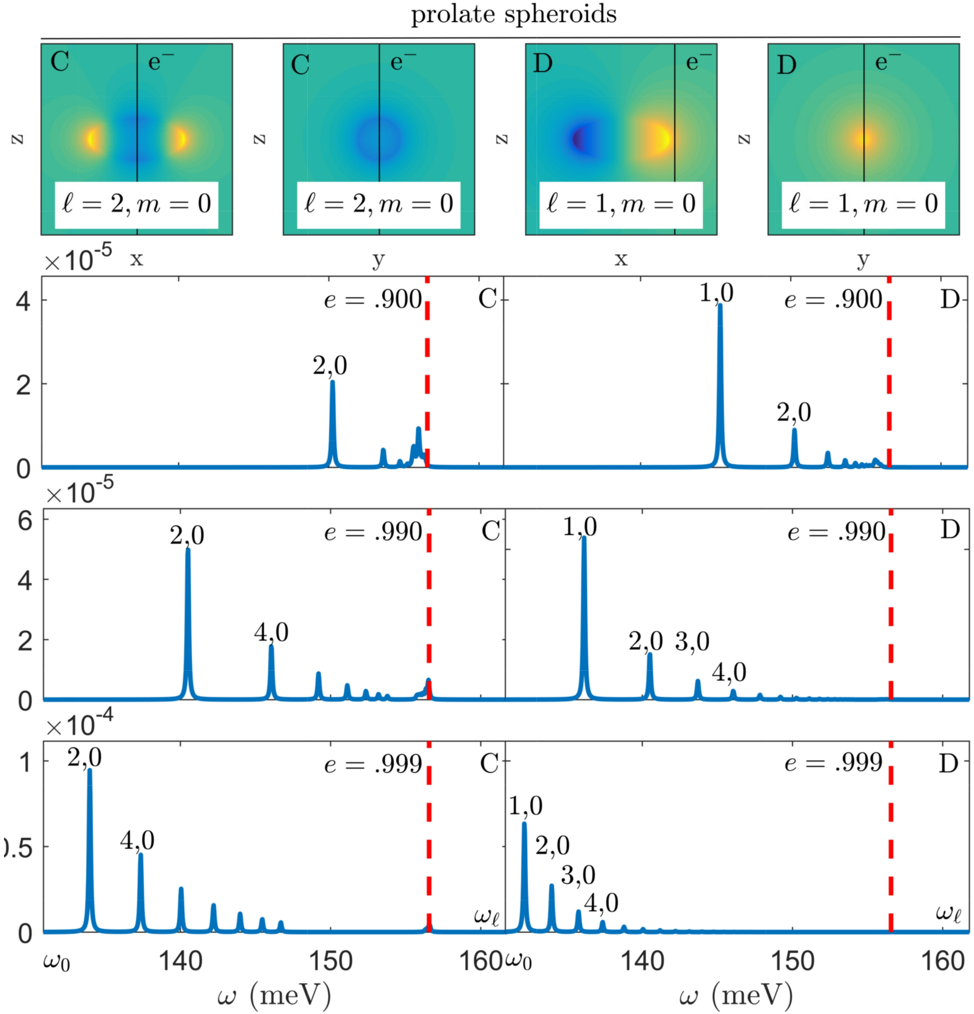
- **States with $\epsilon(\omega)$≈-1**
  - Mid-range energy
  - Weakly scattering
  - High density-of-states

- When $\phi_h^{\text{in}} \approx -\phi_h^{\text{out}}|_{\text{surface}}$, the electric field is determined by local surface charge, and the surfaces are approximately decoupled

Prolate Spheroids as Lightning Rods



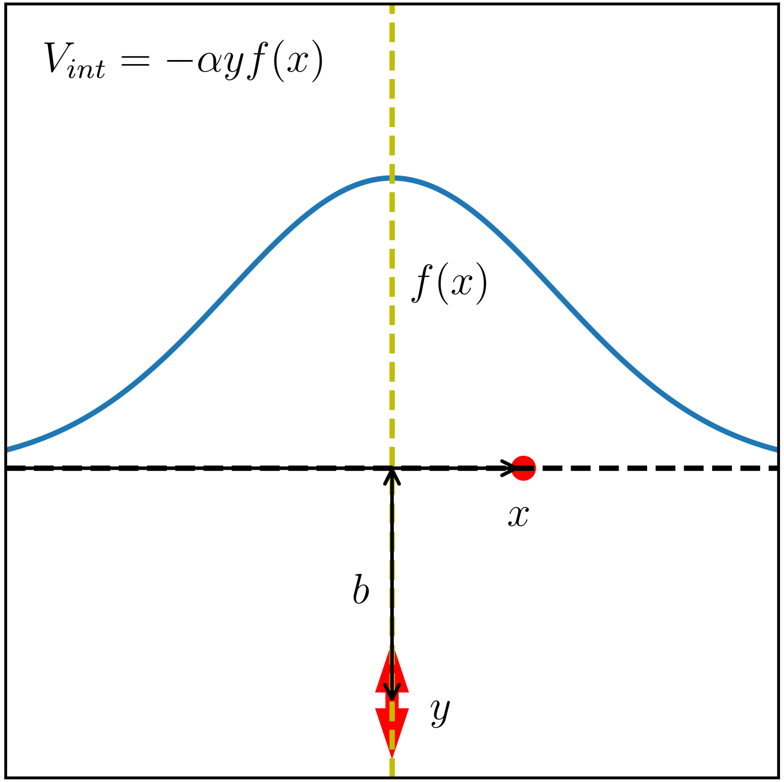
SP mode 1 SP mode 2 SP mode 3



Project 3: Classical-Quantum Comparison

- Let's capture this with a **1D model**:
  - **1D beam** (variable: x, mass: m)
  - **1D oscillator** (variable: y, mass: μ, angular frequency: ω_0)
- The Hamiltonian is straightforward:

$$H = \underbrace{\frac{p_x^2}{2m}}_{\text{beam}} + \underbrace{\frac{p_y^2}{2\mu} + \frac{1}{2}\omega_0^2 \mu y^2}_{\text{oscillator}} - \underbrace{\alpha\, y\, f(x)}_{\text{interaction}}$$

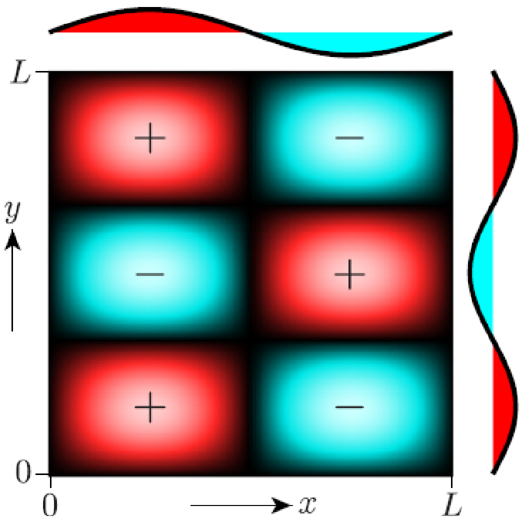


Background Reading

- Explicit illustration: **two 1D particles** trapped in a box
- Example on the right is "**entangled**" – can't factor it!

$$\psi(x,y) \propto \sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)$$

$$\psi \propto \sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{3\pi y}{L}\right) + \frac{1}{2}\sin\left(\frac{3\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)$$



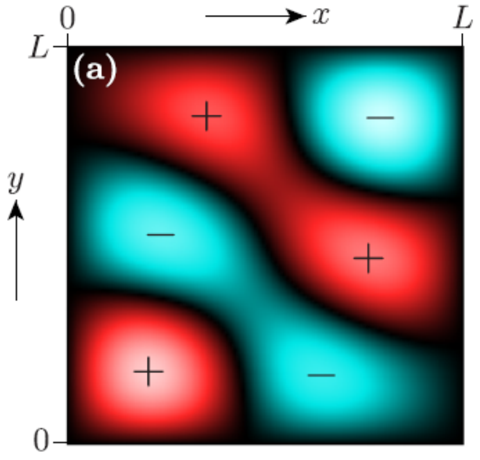
Entanglement isn't just for spin

Daniel V. Schroeder

Check for updates

Am. J. Phys. 85, 812–820 (2017)

https://doi.org/10.1119/1.5003808



Entanglement and Inelastic Scattering

- **Entanglement** in scattering situations arises naturally from the **interaction** between the "beam" and the "target"

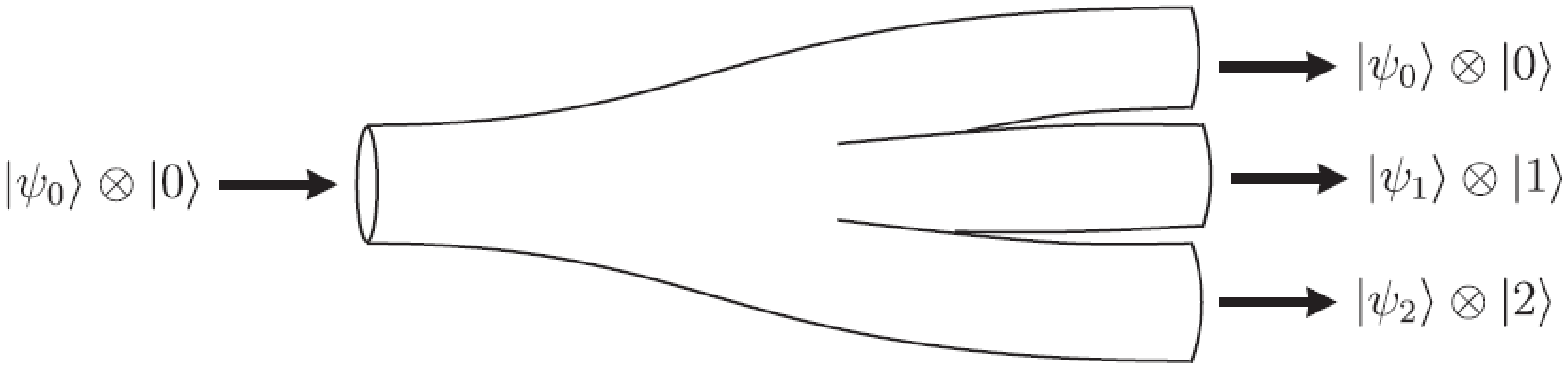


Illustration by Christian Dwyer, in a review article
"Atomic-Resolution Core-Level Spectroscopy
in the Scanning Transmission Electron Microscope"

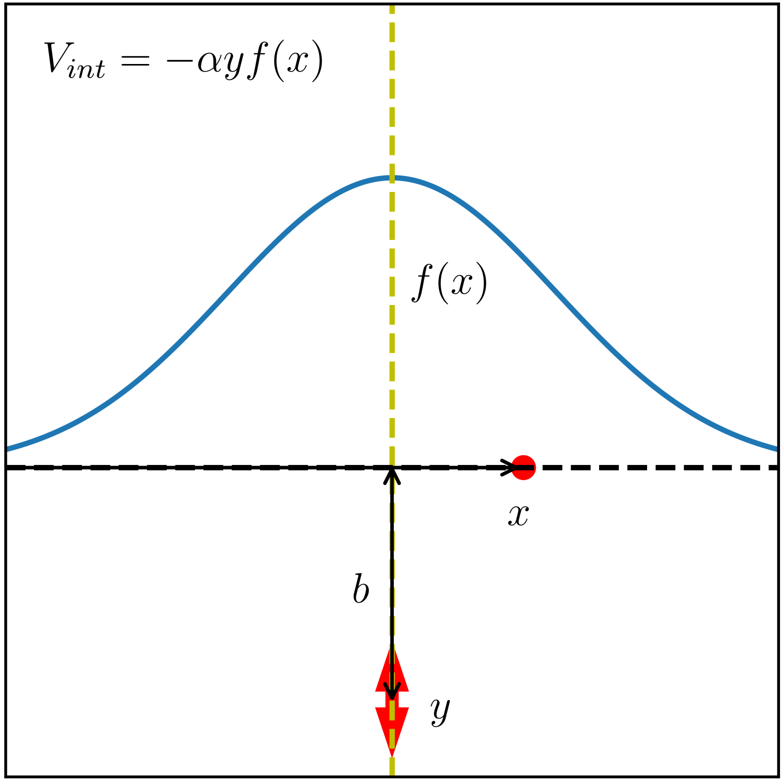
Consequences of the Full Classical Model

- We can find **classical equations of motion** for each variable

$$m\ddot{x} = \alpha y \frac{df}{dx}$$

$$\mu\ddot{y} = -\mu\omega_0^2 y + \alpha f(x)$$

$$H = \underbrace{\frac{p_x^2}{2m}}_{\text{beam}} + \underbrace{\frac{p_y^2}{2\mu} + \frac{1}{2}\omega_0^2\mu y^2}_{\text{oscillator}} - \underbrace{\alpha\, y\, f(x)}_{\text{interaction}}$$



Consequences of the Full Classical Model

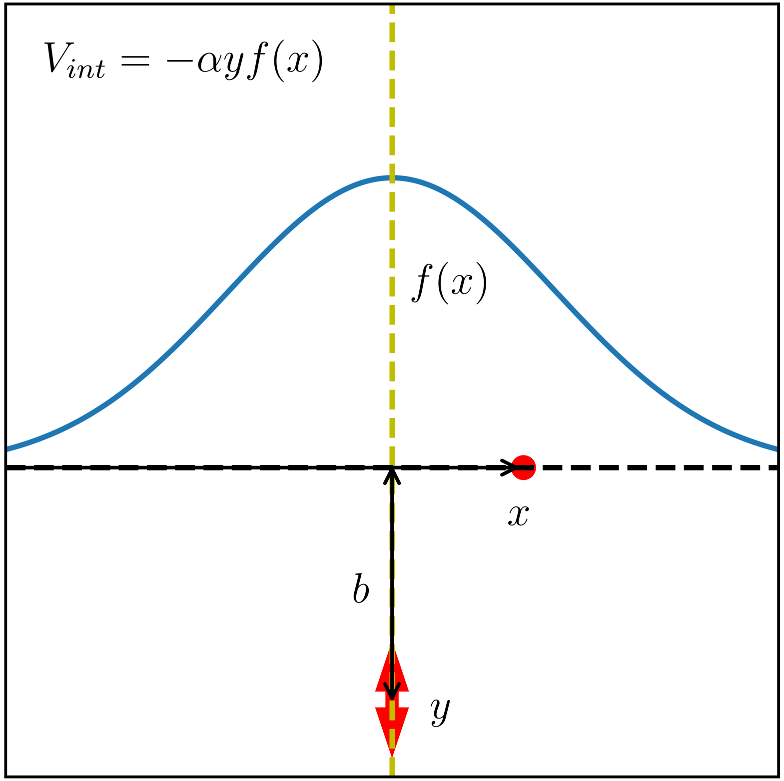
$$H = \underbrace{\frac{p_x^2}{2m}}_{\text{beam}} + \underbrace{\frac{p_y^2}{2\mu} + \frac{1}{2}\omega_0^2 \mu y^2}_{\text{oscillator}} - \underbrace{\alpha\, y\, f(x)}_{\text{interaction}}$$

- We can find classical equations of motion for each variable

$$m\ddot{x} = \alpha y \frac{df}{dx}$$

$$\mu \ddot{y} = -\mu\omega_0^2 y + \alpha f(x)$$

- When the beam travels swiftly ($x \approx v_0 t$), a **predictable amount of energy is transferred** from beam to oscillator (assuming it starts at rest)



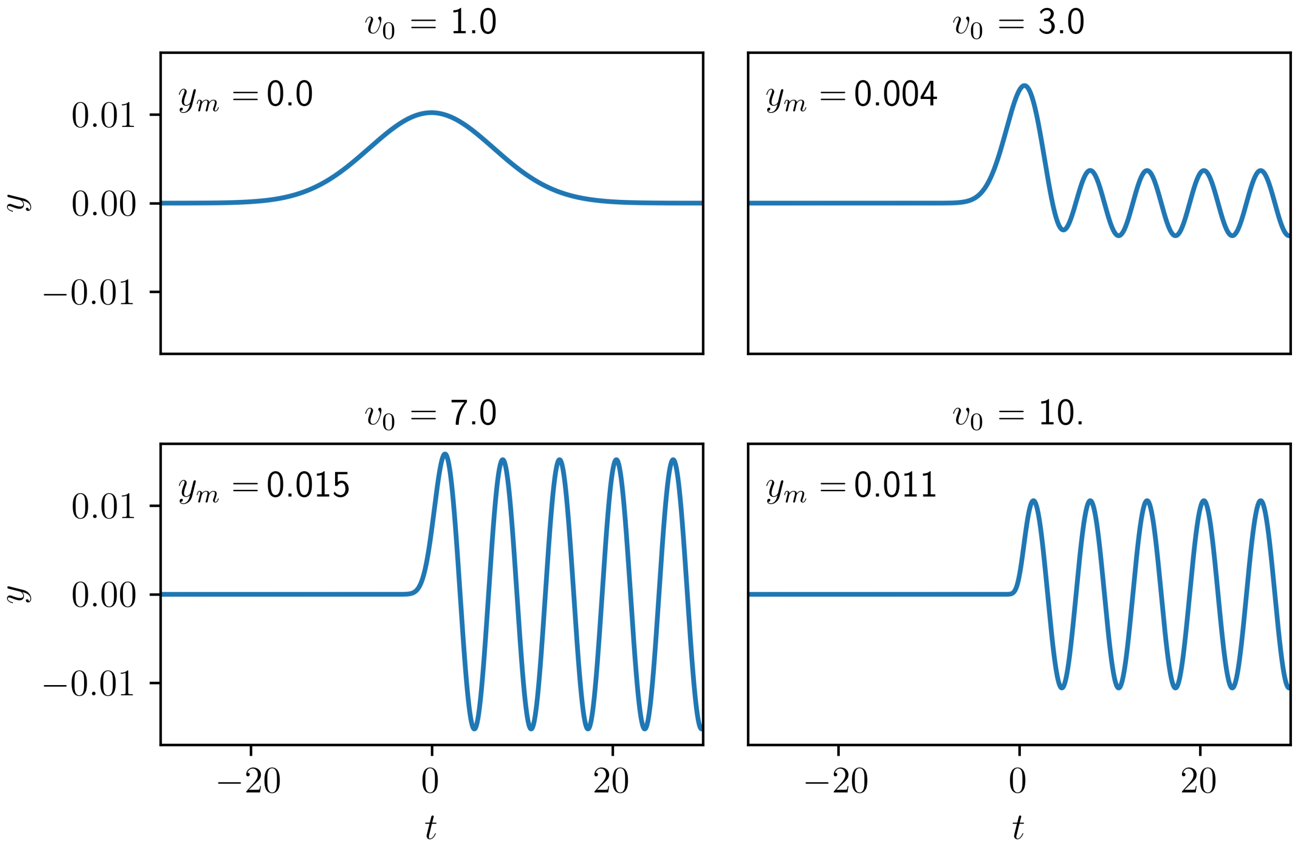
Classical Oscillations

- Final **amplitude** depends directly on **model** and **initial conditions**

$$m\ddot{x} = \alpha y \frac{df}{dx}$$

$$\mu\ddot{y} = -\mu\omega_0^2 y + \alpha f(x)$$

$$f(x) = b^{-2} e^{-x^2/b^2}$$



Quantum Harmonic Oscillator

- States of the QHO look like **trapped waves**

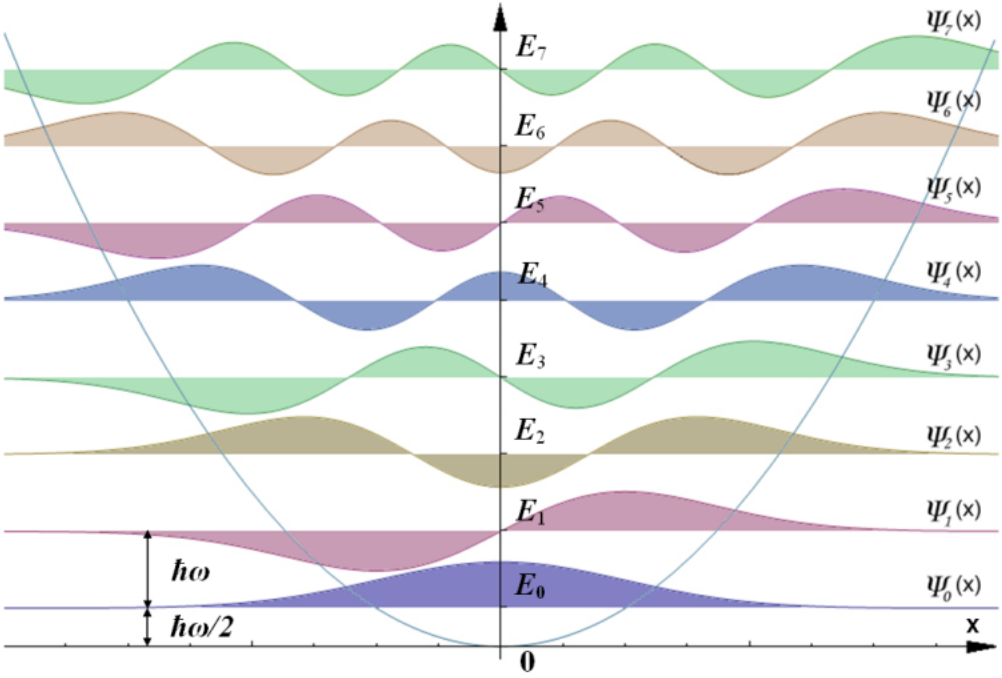
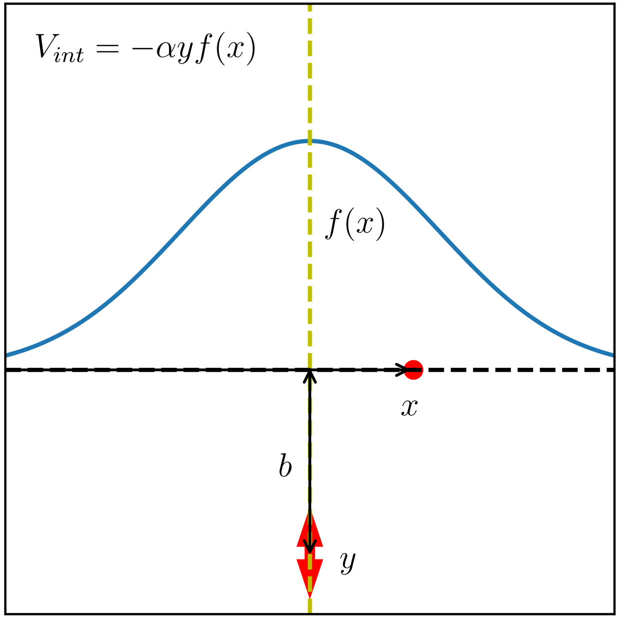


Image from Wikipedia

Quantizing Just the Oscillator

- Problem: Classical approach doesn't reflect experiment – **experiment only has *some* beam particles losing energy**
- Trial fix: Use a **classical beam**, and a **quantum oscillator**

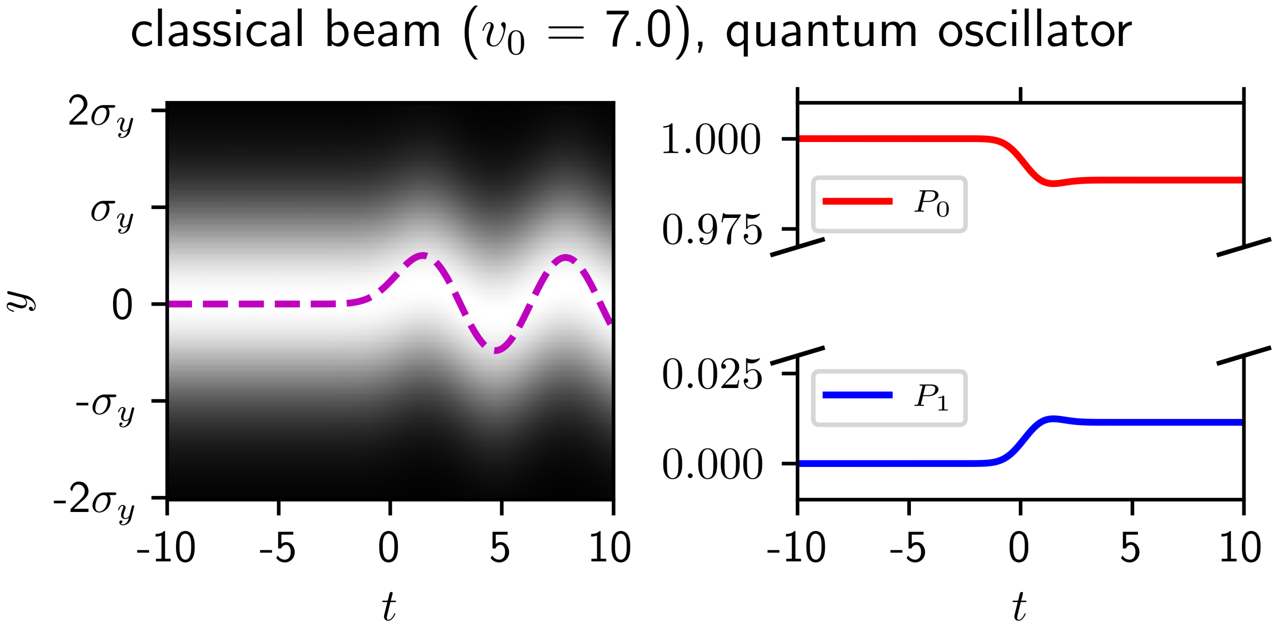
$$H = \underbrace{\frac{p_y^2}{2\mu} + \frac{1}{2}\omega_0^2 \mu y^2}_{\text{unperturbed oscillator}} - \underbrace{\alpha y f(vt)}_{\text{time-dependent perturbation}}$$



Quantizing Just the Oscillator

- We can numerically integrate the Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}\psi(y,t) = \hat{H}(t)\psi(y,t)$$



$$\langle y\rangle = \int dy\, \psi^*(y)\, y\, \psi(y)$$

$$\langle\psi_1|\psi\rangle = \int dy\, \psi_1^*(y)\psi(y)$$

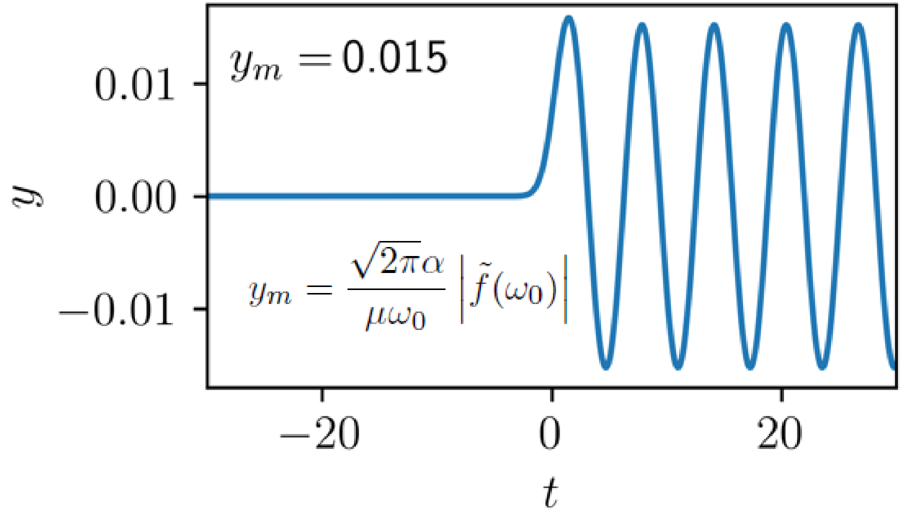
$$P_1 = |\langle\psi_1|\psi\rangle|^2$$

Classical vs. Partly Quantum Approaches

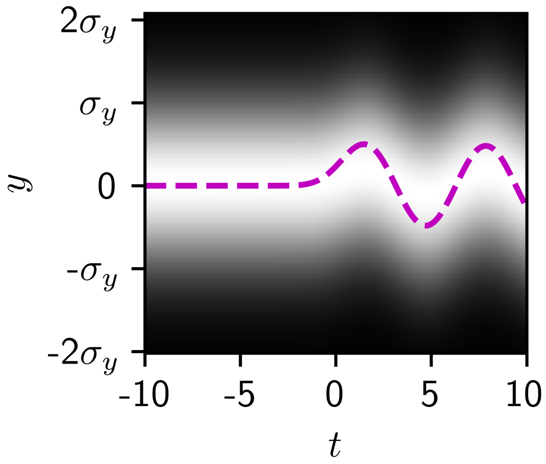
- The classical amplitude of the classical HO oscillator is proportional to the transition amplitude of the quantum HO

$$y_m = \sqrt{\frac{2\hbar}{\mu\omega} P_1^{(1)}} \qquad\qquad P_1^{(1)} = \frac{\mu\omega_0}{2\hbar} y_m^2$$

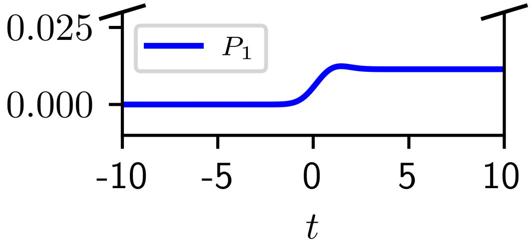
$v_0 = 7.0$



classical beam ($v_0 = 7.0$), quantum oscillator



$$P_1 = \frac{\pi\alpha^2}{\hbar\mu\omega_0}|\tilde{f}(\omega_0)|^2$$



Partly QM vs. Fully QM Approaches

- When will the **partly QM** approach match the fully QM one?

Partly QM Approach

$$i\hbar\frac{\partial}{\partial t}\psi(y) = \hat{H}(t)\psi(y)$$

$$\hat{H}(t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial y^2} + \frac{1}{2}\mu\omega_o^2 y^2 - \alpha y f(vt)$$

Partly QM vs. Fully QM Approaches

- When will the **partly QM** approach match the **fully QM** one?

Partly QM Approach

$$i\hbar\frac{\partial}{\partial t}\psi(y) = \hat{H}(t)\psi(y)$$

$$\hat{H}(t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial y^2} + \frac{1}{2}\mu\omega_o^2 y^2 - \alpha y f(vt)$$

Fully QM Approach

$$i\hbar\frac{\partial}{\partial t}\psi(x,y) = \hat{H}\psi(x,y)$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial y^2} + \frac{1}{2}\mu\omega_o^2 y^2 - \alpha y f(x)$$

Partly QM vs. Fully QM Approaches

- When will the partly QM approach match the fully QM one?

Partly QM Approach

$$i\hbar\frac{\partial}{\partial t}\psi(y) = \hat{H}(t)\psi(y)$$

$$\hat{H}(t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial y^2} + \frac{1}{2}\mu\omega_o^2 y^2 - \alpha y f(vt)$$

Fully QM Approach

$$i\hbar\frac{\partial}{\partial t}\psi(x,y) = \hat{H}\psi(x,y)$$

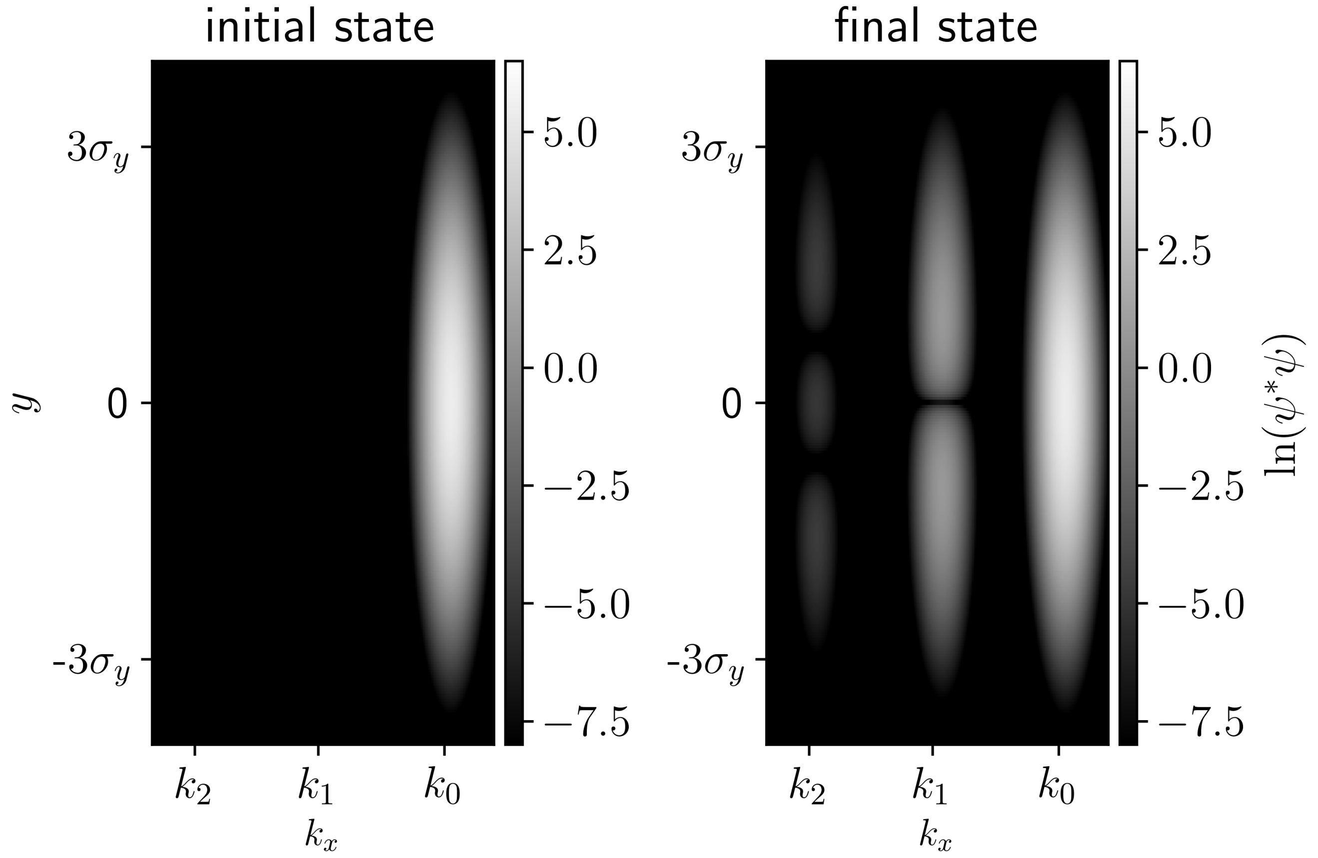
$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial y^2} + \frac{1}{2}\mu\omega_o^2 y^2 - \alpha y f(x)$$

- Depends on the **relation between reduced and full wavefunction**

$$\psi(y) = \int dx\,\psi(x,y)$$

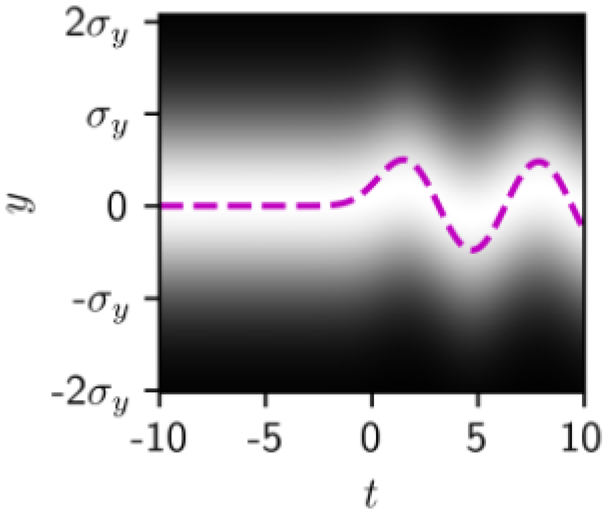
Imagining $\Psi^*\Psi$ in k_x and y

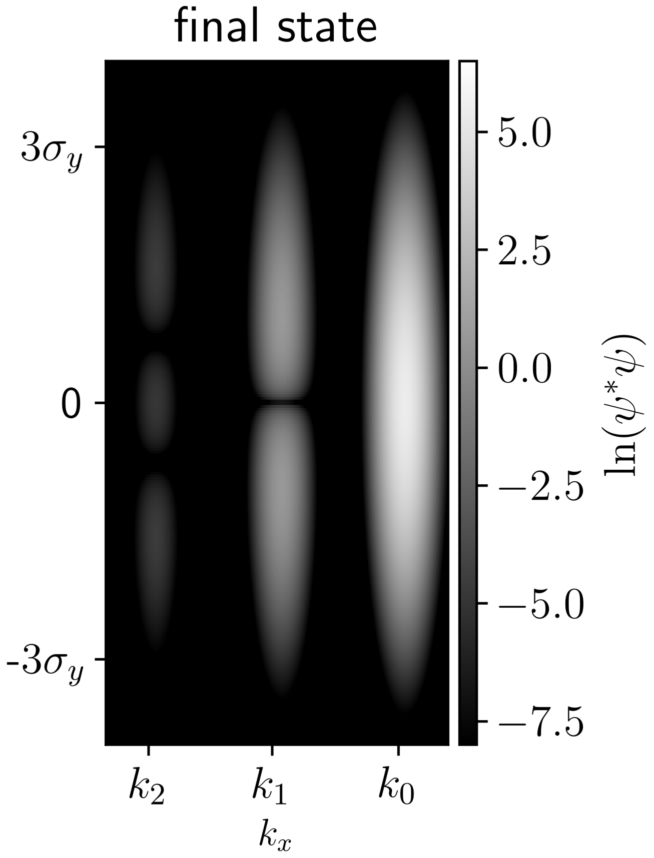
- The probability density of the **beam is entangled with** that of the **oscillator**



Imagining $\Psi^*\Psi$ in k_x and y

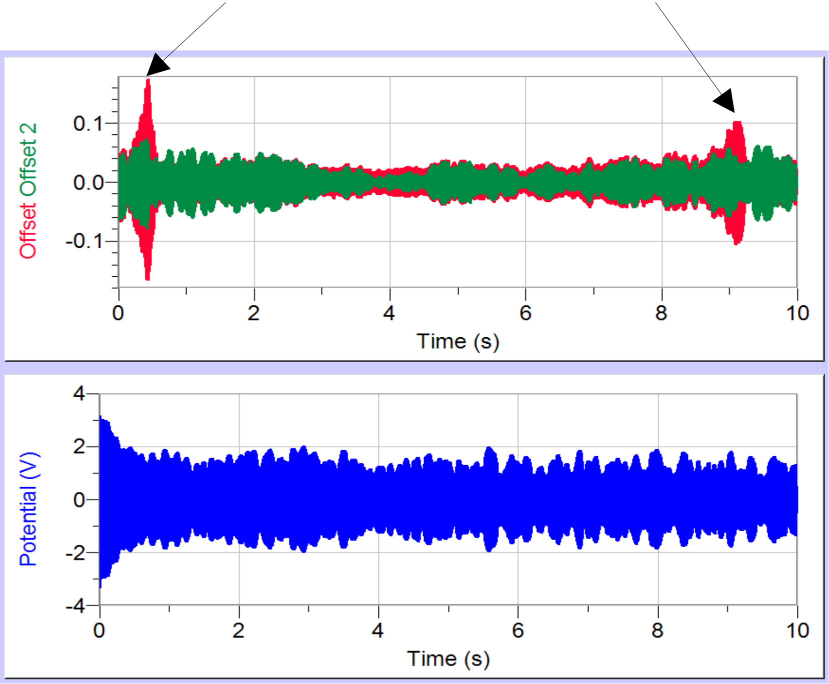
- The probability density of the beam is entangled with that of the oscillator
- **Thinking question**: how does this static density relate to the time-dependence of the reduced $\Psi(y)$?

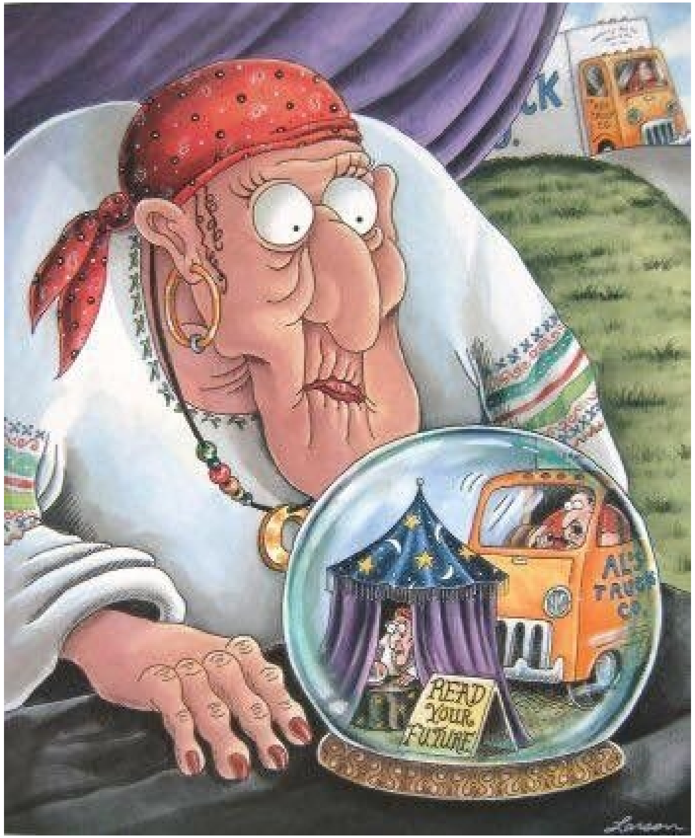




Future Projects?

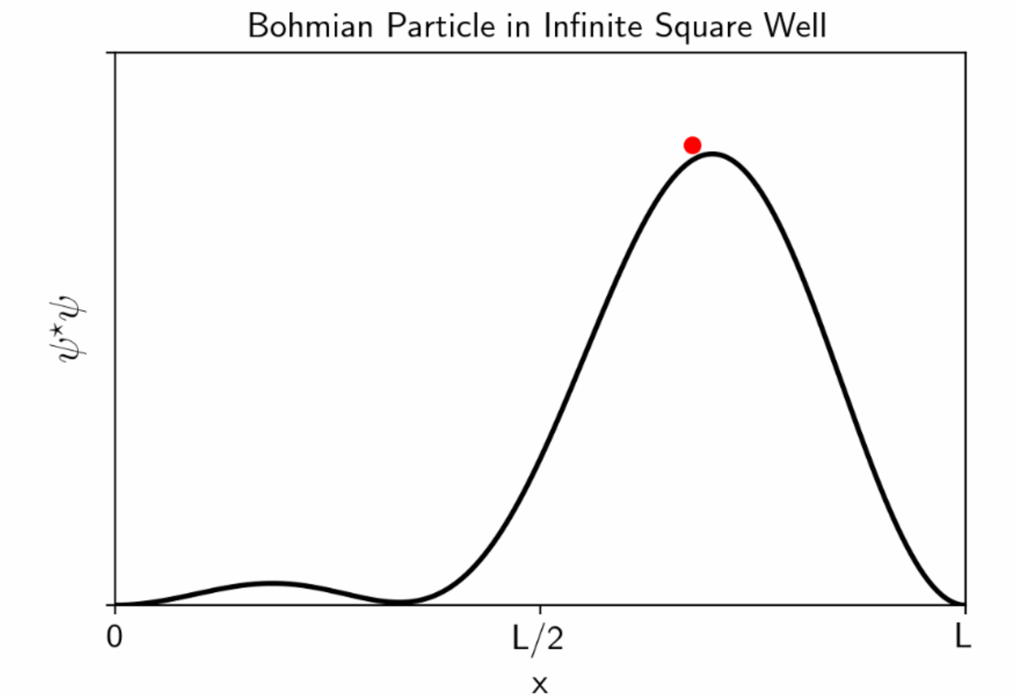
- Lots of acoustical possibilities – from acoustical metamaterials to taking oscillator descriptions in a quantum direction

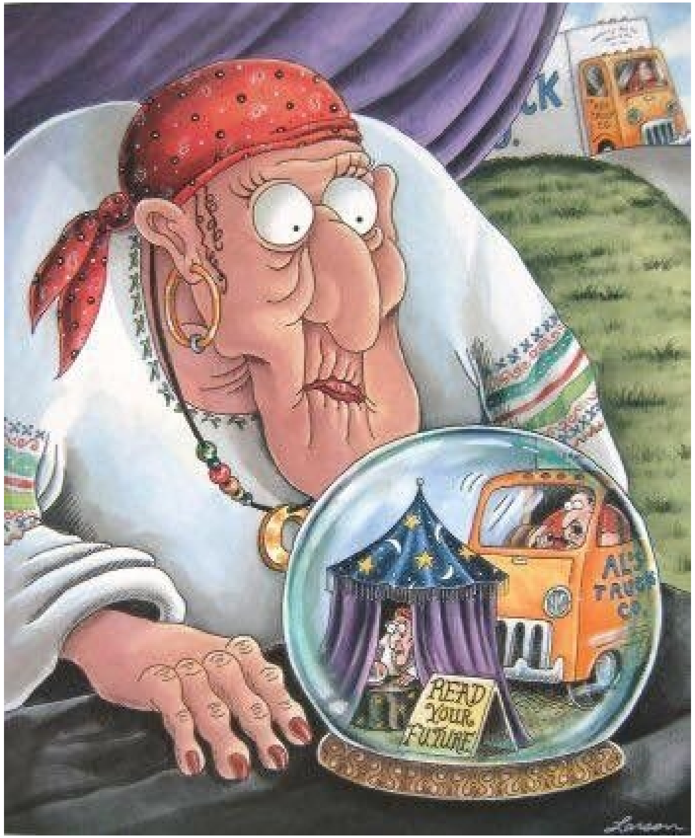




Future Projects?

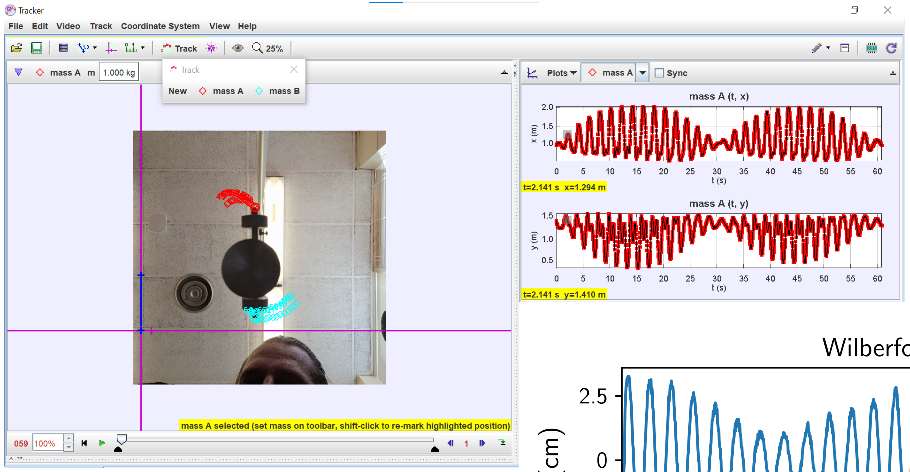
- Quite a few "quantum fun" projects – from STEM-EELS modeling to exploring questions of quantum foundations

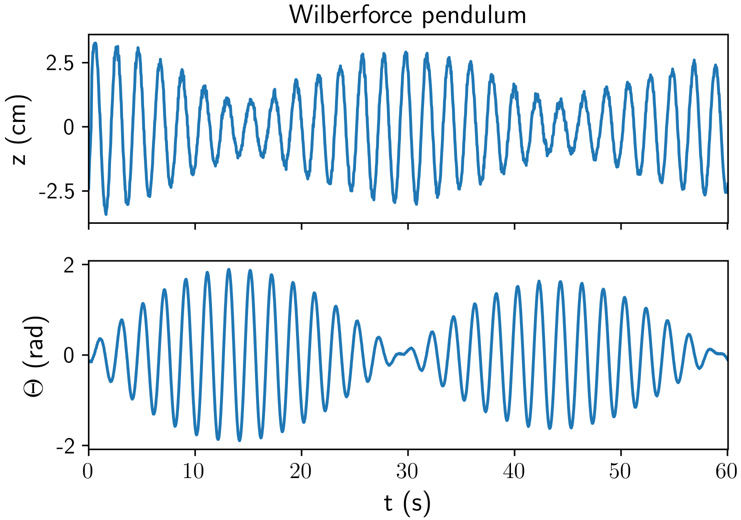


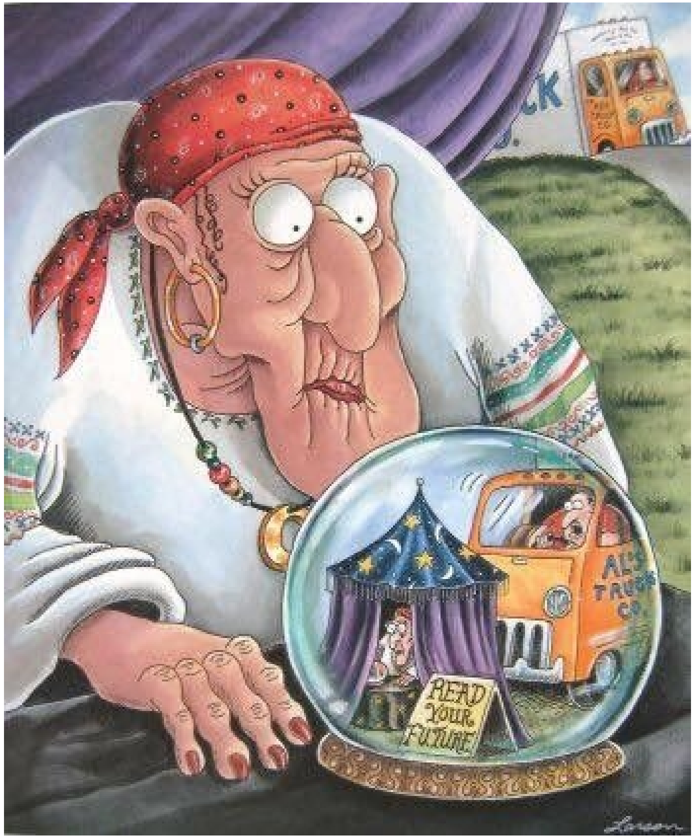


Future Projects?

- Video analysis with students is always fun!







Summary

- Same mathematical tools can often be used across many different physical scenarios
- Driven oscillators are a good description for many different oscilliatory phenomena
- The classical and quantum versions of such models rhyme, but aren't the same