Cherenkov Excitation of Waveguide Modes in the Electron Microscope

Dr. David Kordahl
November 6, 2020

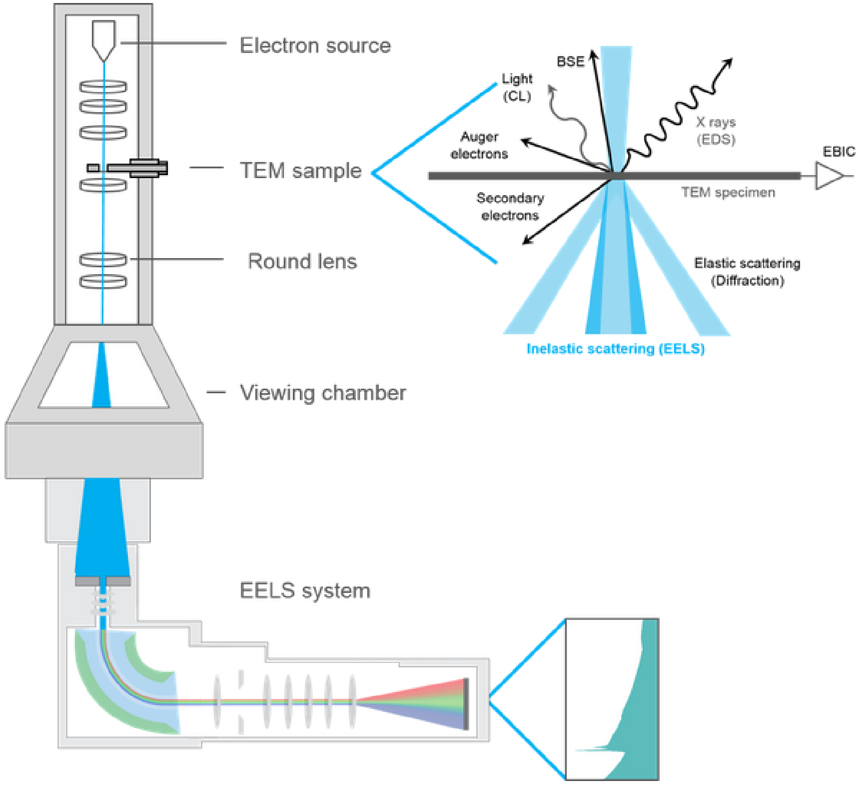
Centenary College of Louisiana
Arizona State University

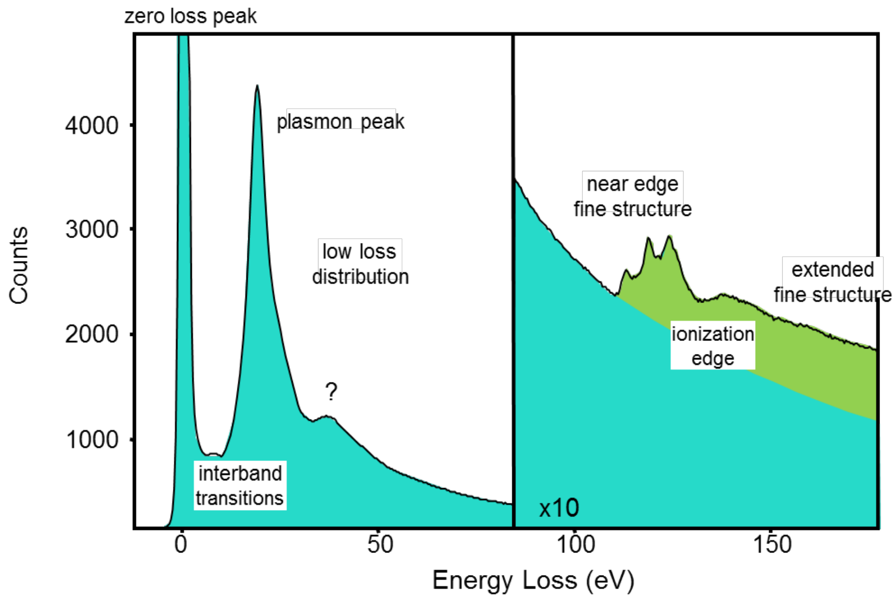
Outline

- Basic Principles of STEM-EELS
- Physics of the Cherenkov Effect
- Physics of Waveguide Modes
- STEM-EELS Excitation of Waveguide Modes in Ribbons
- STEM-EELS Excitation of Waveguide Modes in Discs

Introducing STEM-EELS

STEM = Scanning Transmission Electron Microscope



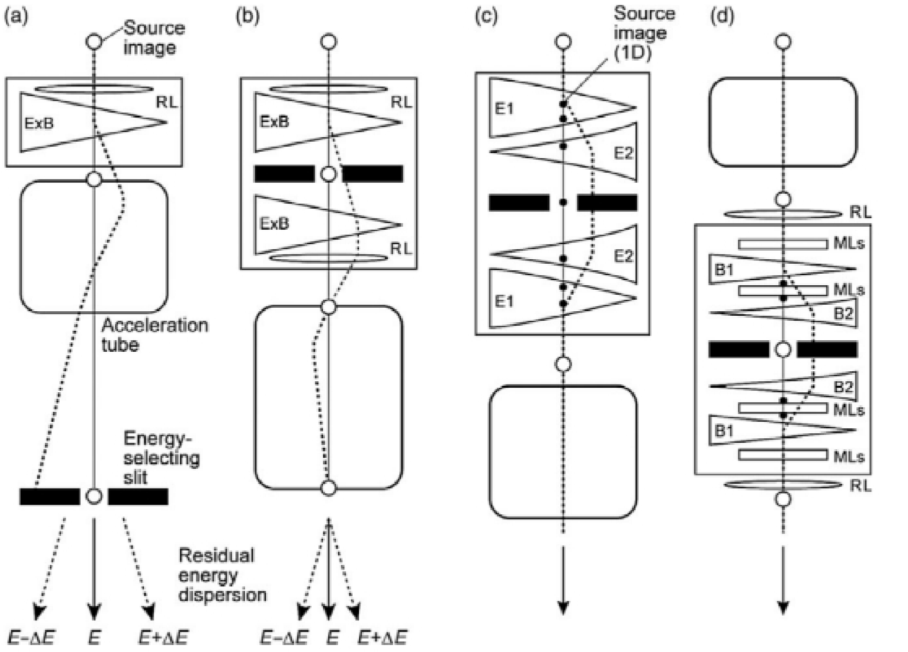


EELS = Electron Energy-Loss Spectroscopy

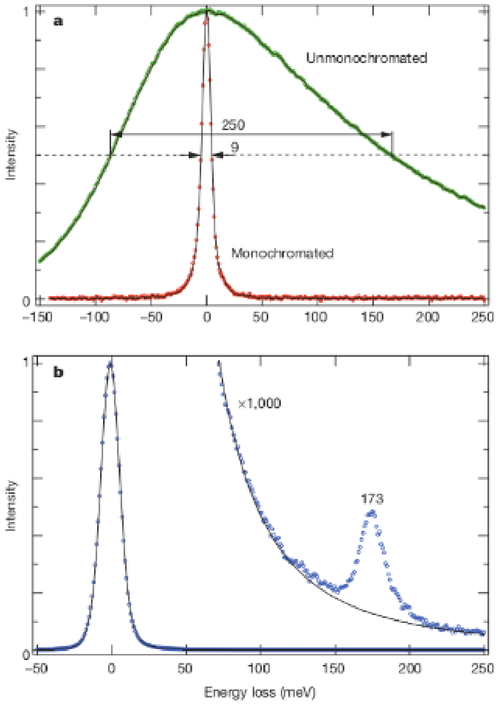
https://eels.info/about/overview

Improved Energy Resolutions in STEM-EELS

“Practical aspects of mono-chromators developed for transmission electron microscopy,” K. Kimoto, Microscopy, 337–344 (2014).



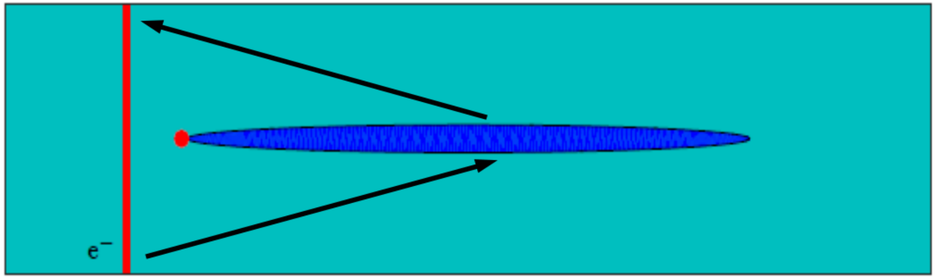
“Vibrational spectroscopy in the electron microscope,” O. L. Krivanek, et al., Nature 514, 209 (2014).



Classical and Quantum Approaches to EELS

In the classical approach, the induced electric field from the particle pushes back on the beam, causing energy transfer:

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} = \frac{e}{\pi\hbar\omega}\int \mathrm{d}z \mathrm{Re}\big[E^z(\mathbf{x},z)\exp(-i\omega z/v)\big]$$



In the quantum approach, modes of the dielectric sample are the quantum states, and transition probabilities are found:

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} = \sum_n \frac{\eta}{2\pi}\frac{P_n}{(\omega-\omega'_n)^2+(\eta/2)^2}$$

where, at a beam position **x**,

$$P_n = \left(\frac{e}{\hbar\omega_n}\right)^2\left|\int E_n^z(\mathbf{x},z)\exp(i\omega_n z/v)dz\right|^2$$

Simple Picture of Cerenkov Radiation

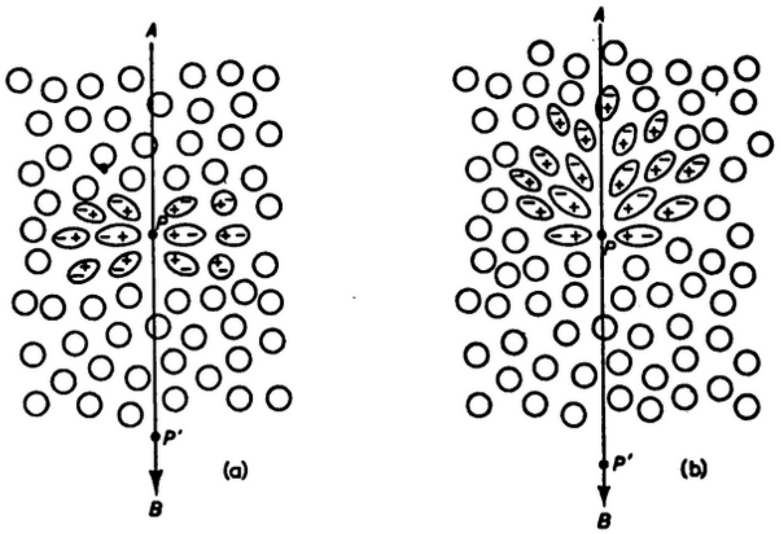
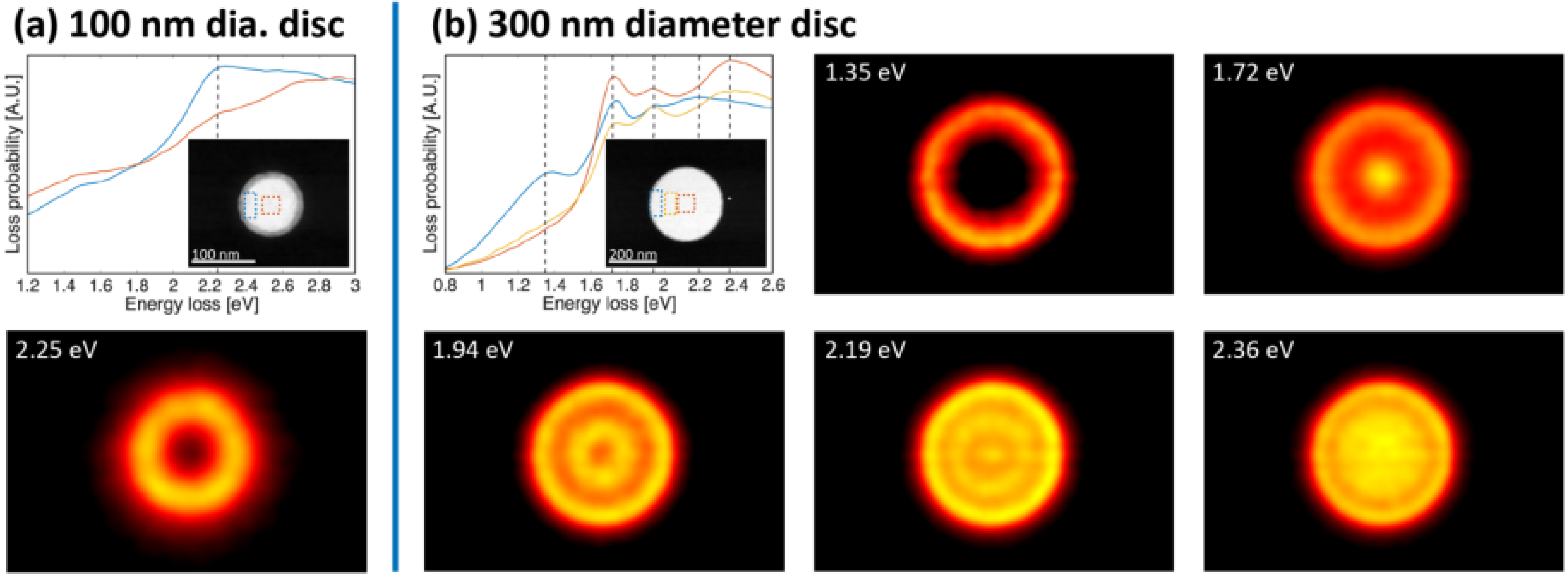


FIG. 1.1. The polarization set up in a dielectric by the passage of a charged particle. (a) At low velocity. (b) At high velocity.

- Jelley, J. V., “Cerenkov radiation and its applications”, British Journal of Applied Physics 6, 7, 227–232 (1955).

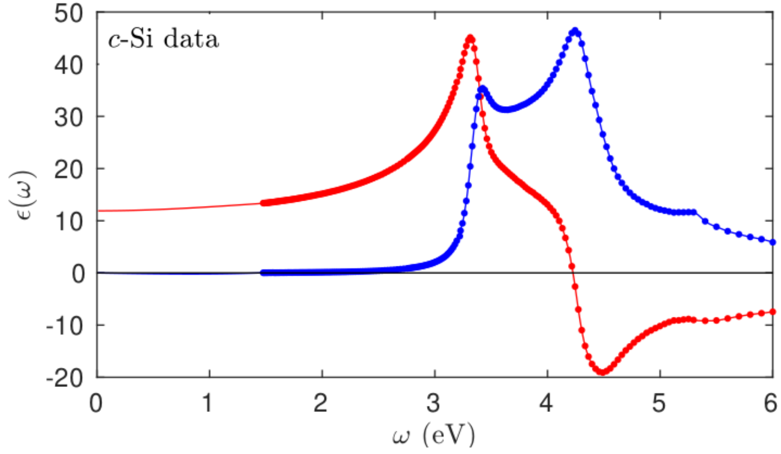
Cherenkov Physics in Silicon Discs?

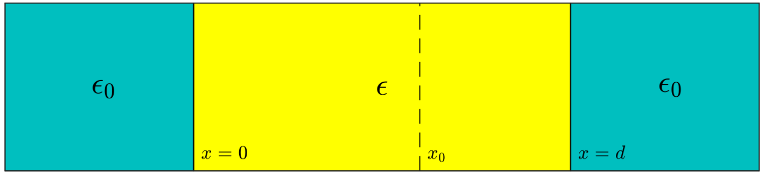
Resonant modes have been imaged in thick (100 nm) silicon cylinders by V. Flauraud and D.T.L. Alexander

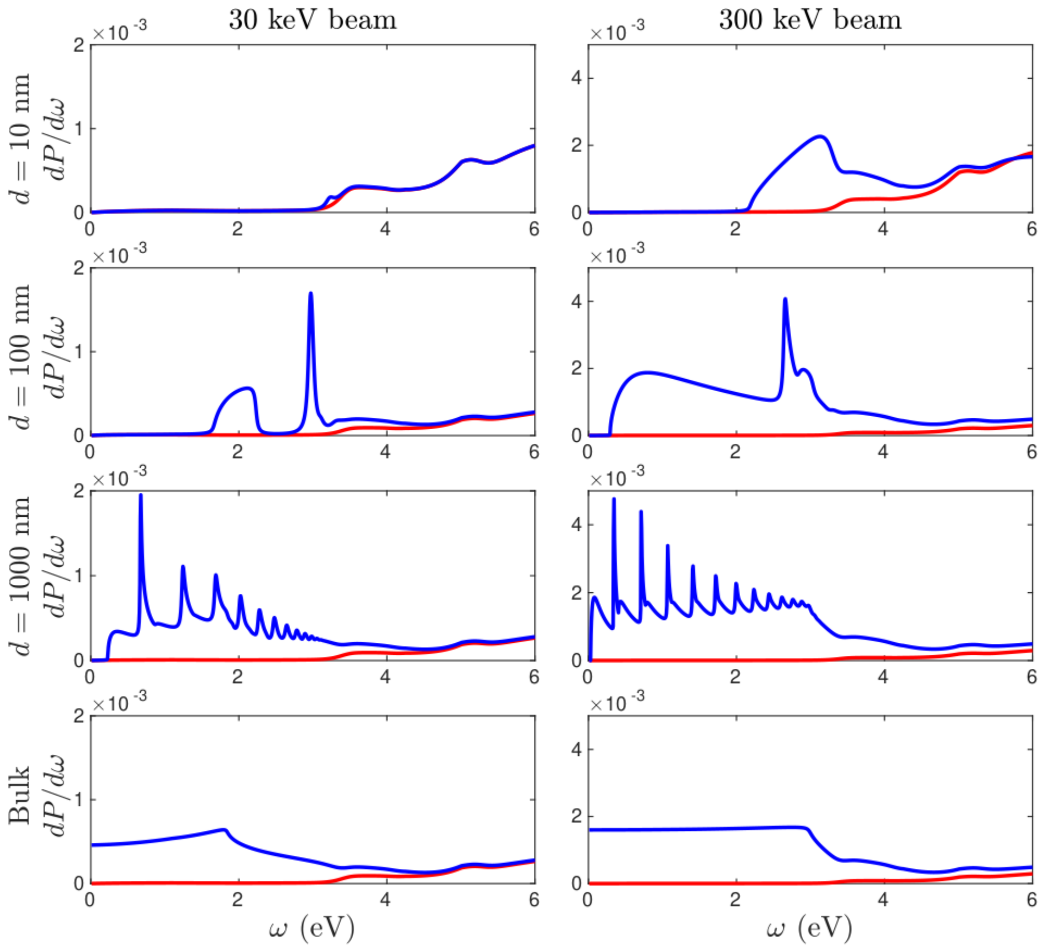


Microsc. Microanal. 25 (Suppl 2), 2019, pp. 364-365

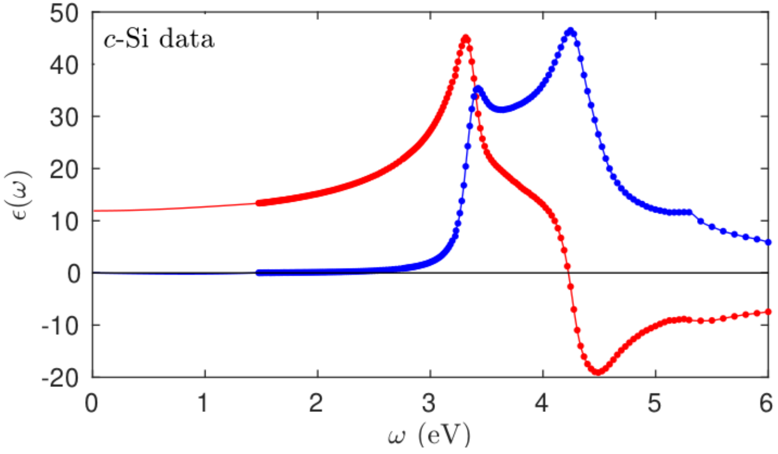
Waveguide Modes in a Dielectric Ribbon

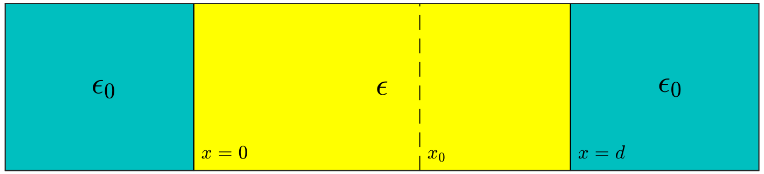






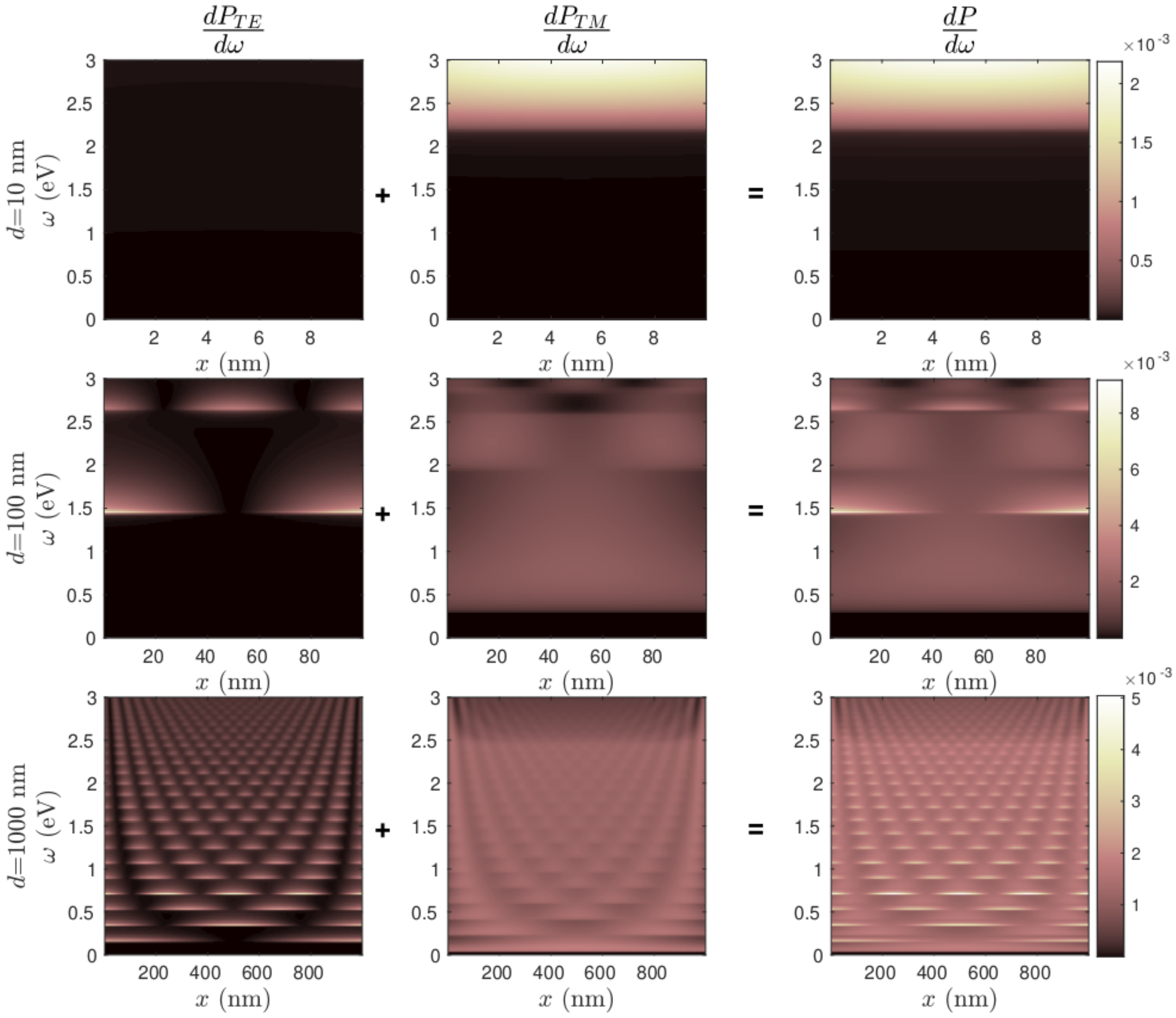
Waveguide Modes in a Dielectric Ribbon





$E_x = 0,\ \frac{E_z}{E_y} \approx -\frac{v k_y}{\omega}$

$H_x = 0,\ \frac{E_z}{E_y} \approx \frac{\omega}{v k_y}$



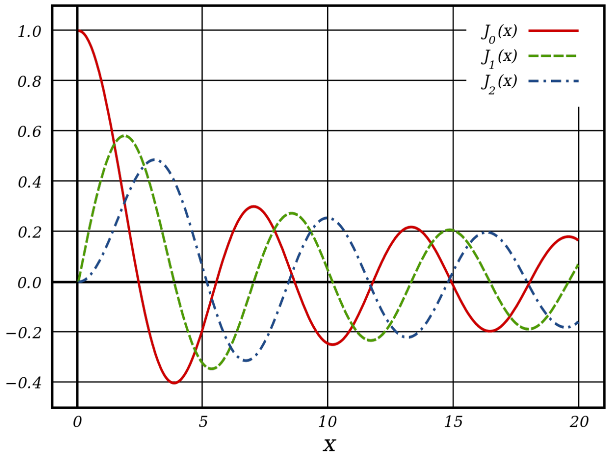
Waveguide Modes in a Dielectric Cylinder

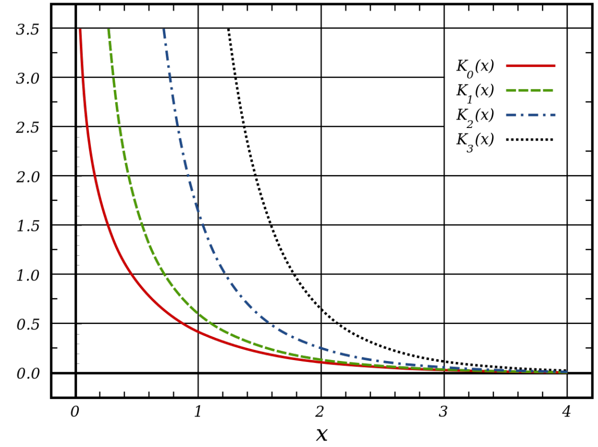
- Inside the cylinder,

$$E_z,\, H_z \propto J_m(k_r^{\text{in}} r)$$

- Outside the cylinder,

$$E_z,\, H_z \propto K_m(k_r^{\text{out}} r)$$

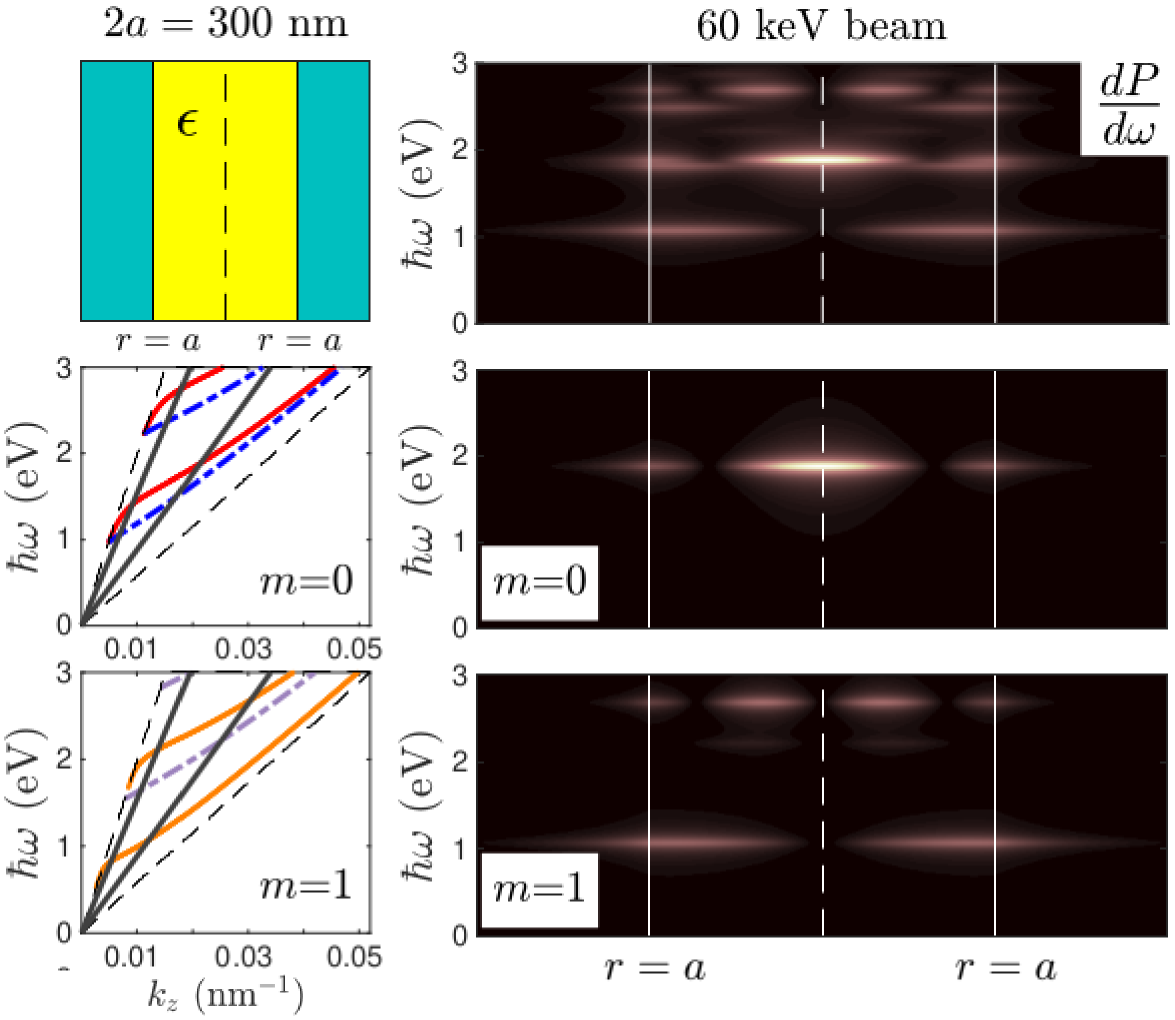




Waveguide Modes in a Dielectric Cylinder

- Imaged modes:
  $\omega_{\ell mn} = v k_z$

- Different modes with similar qualities are imaged at different beam energies

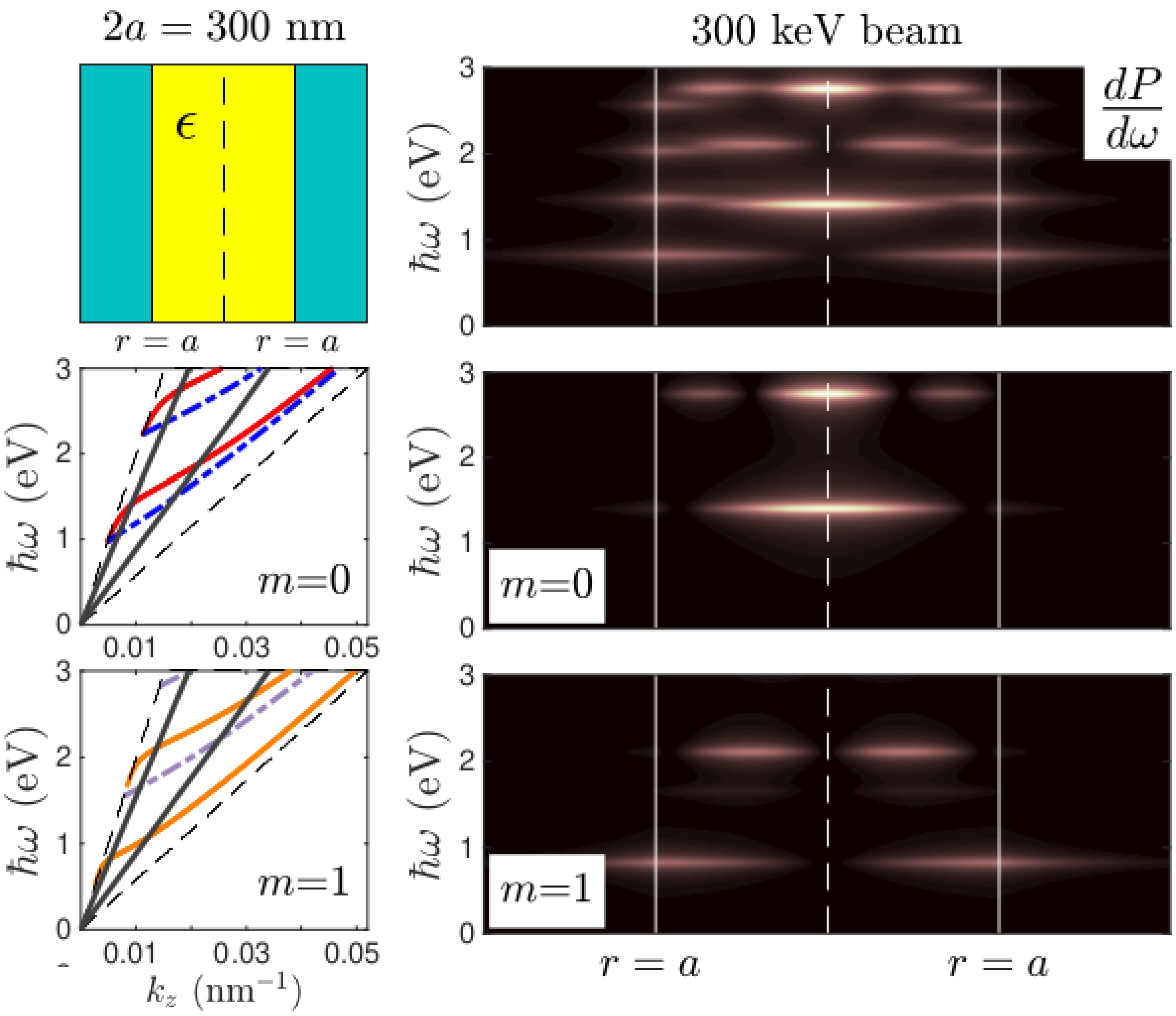


Waveguide Modes in a Dielectric Cylinder

- Imaged modes:

$$\omega_{\ell mn} = vk_z$$

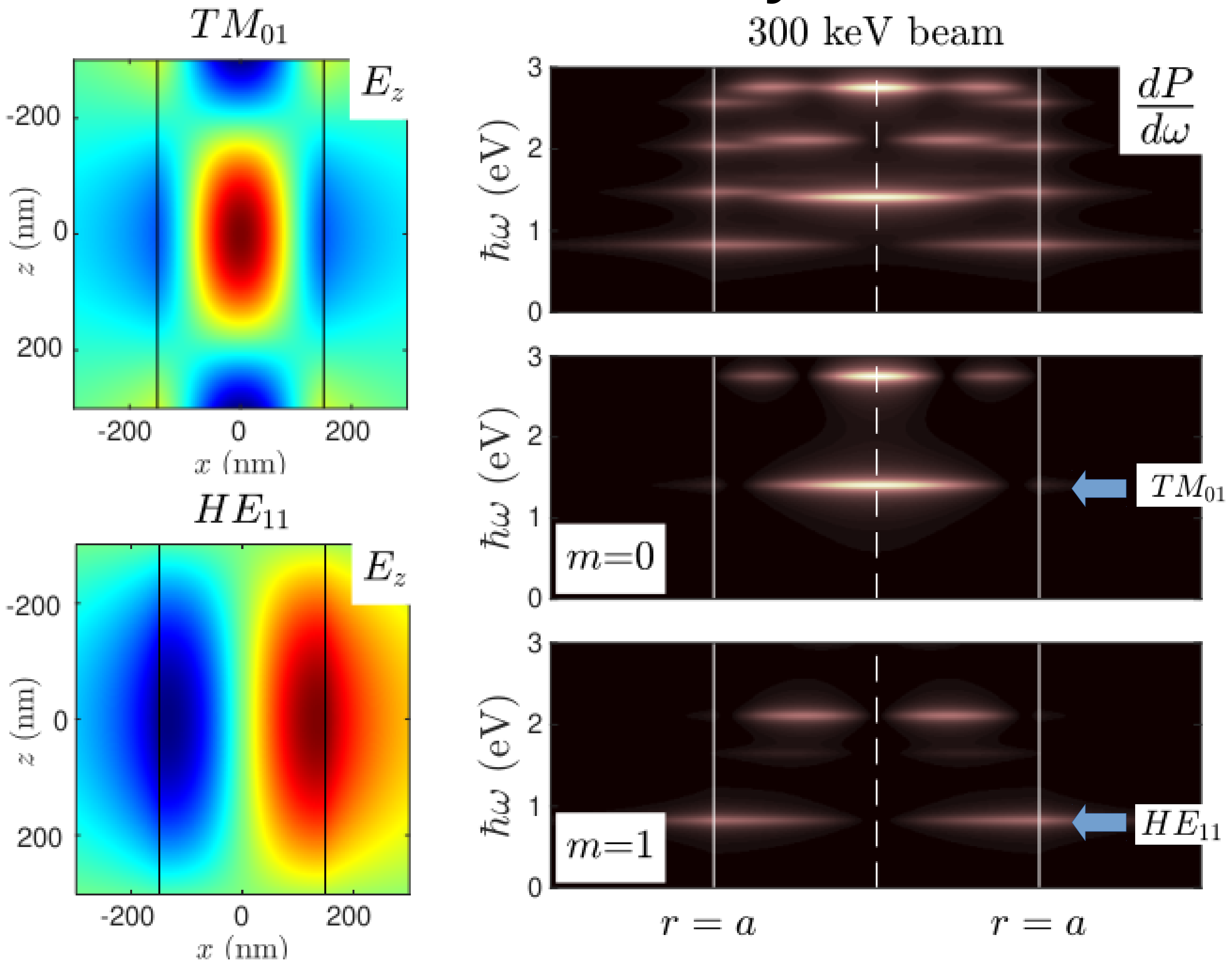
- Different modes with similar qualities are imaged at different beam energies



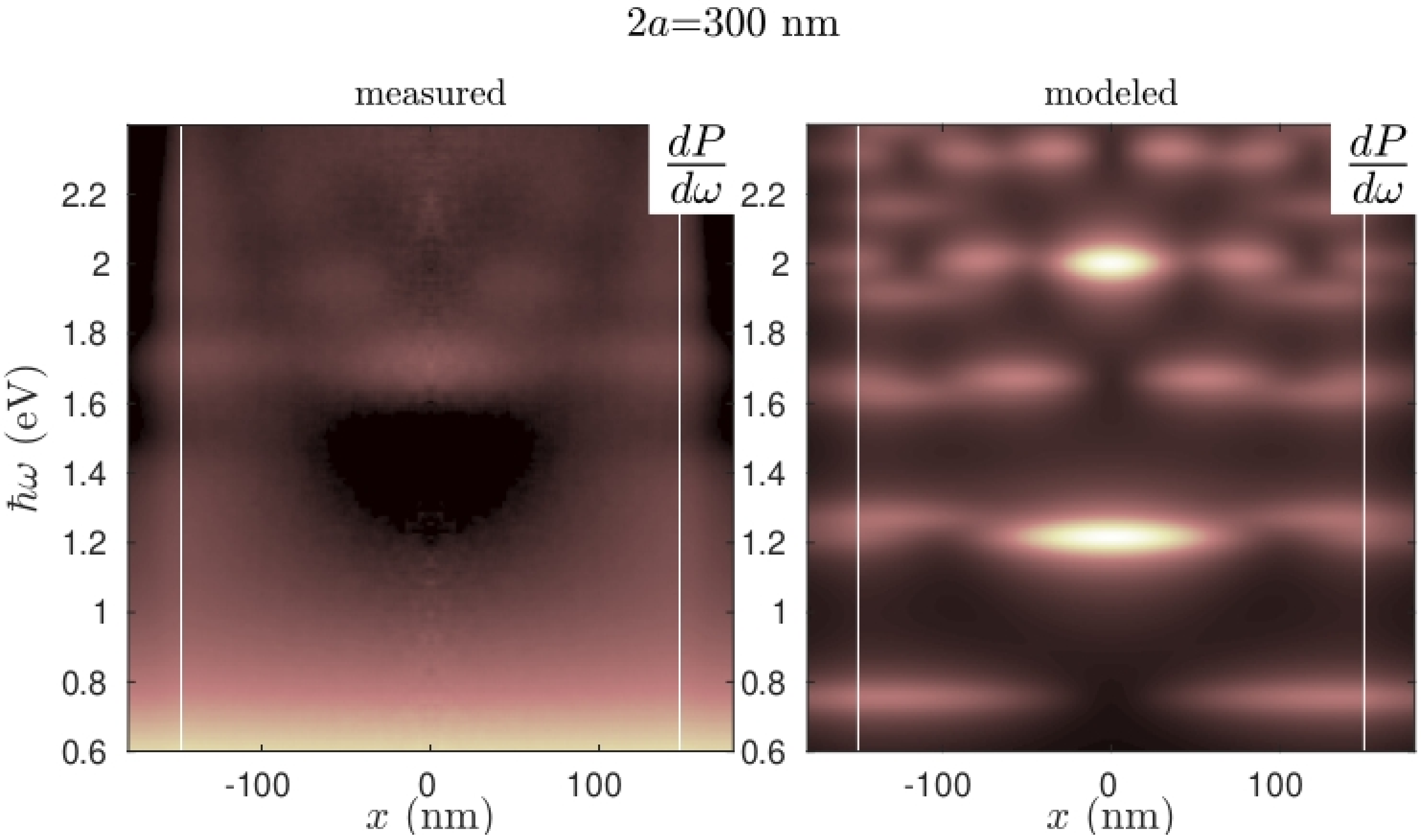
Waveguide Modes in a Dielectric Cylinder

- Imaged modes:
  $$\omega_{\ell mn} = v k_z$$

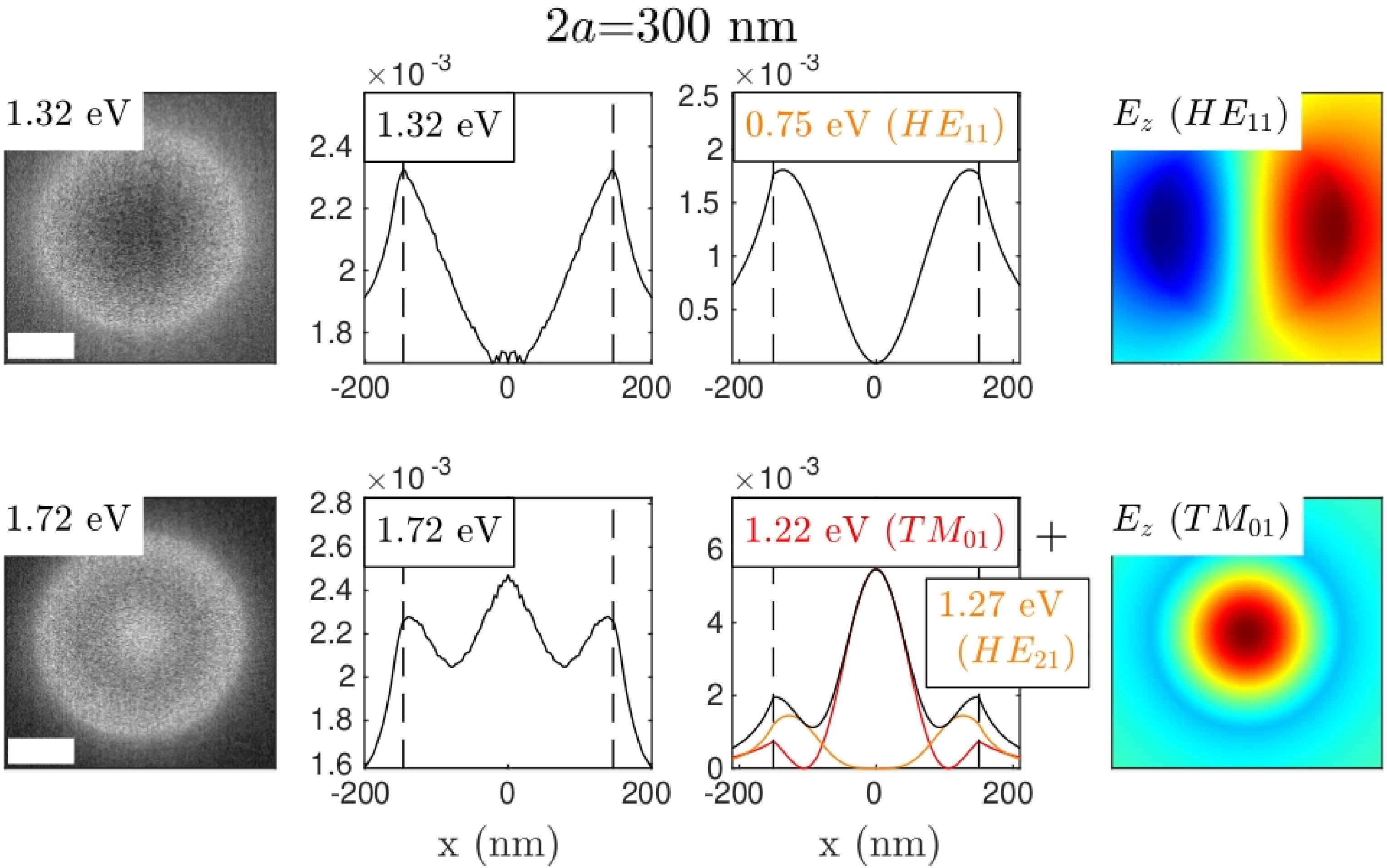
- Different modes with similar qualities are imaged at different beam energies



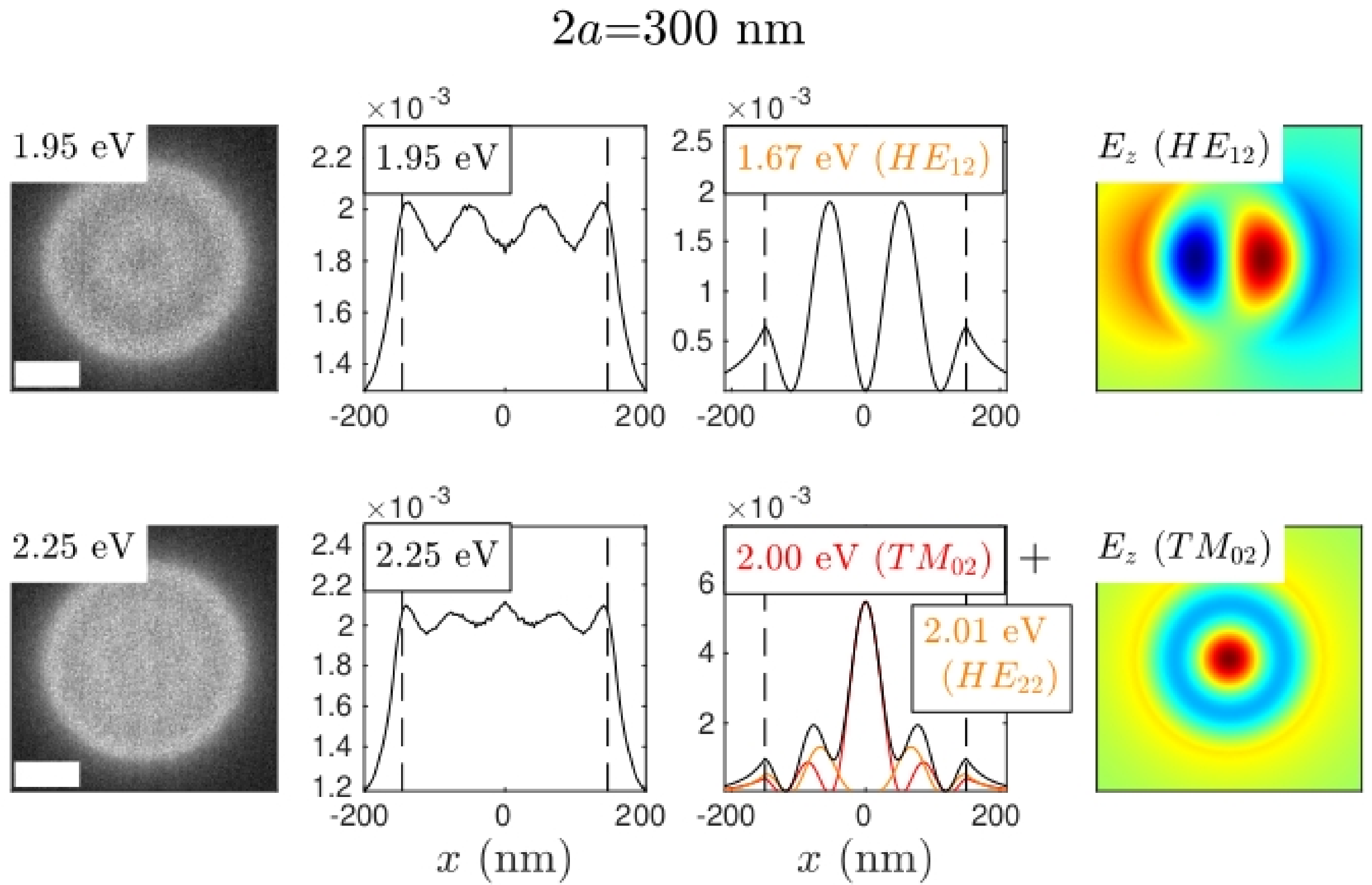
Comparison with Silicon Discs



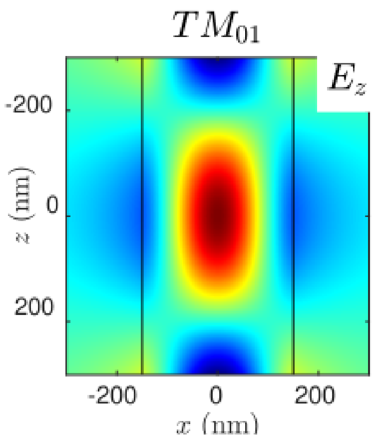
Approximate Modes in Dielectric Discs

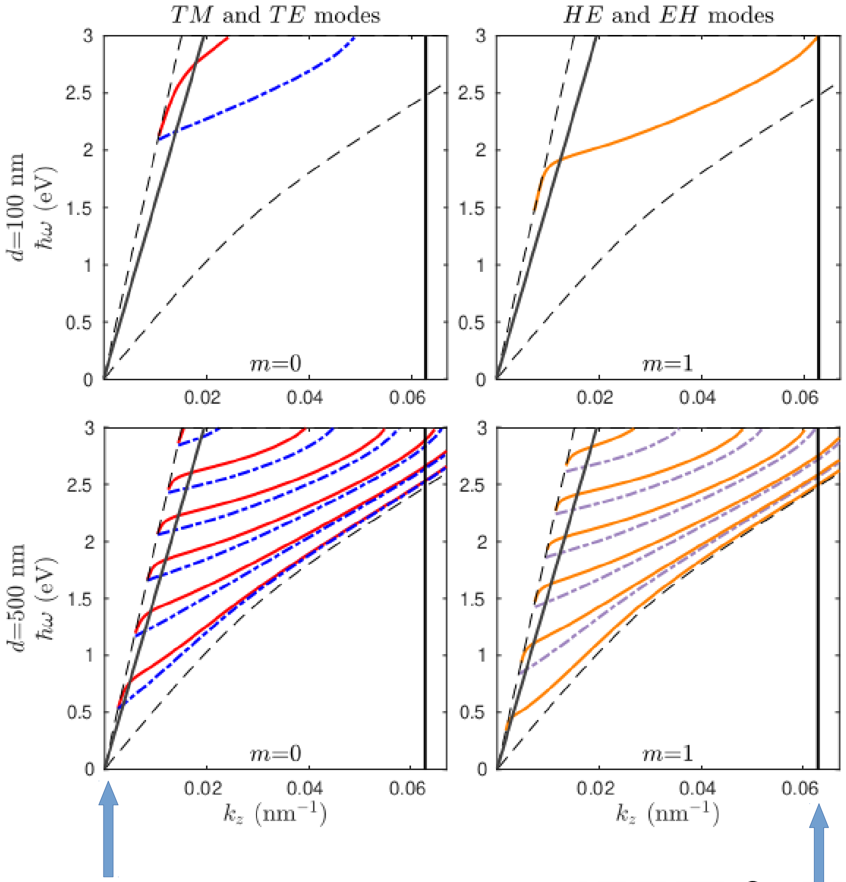


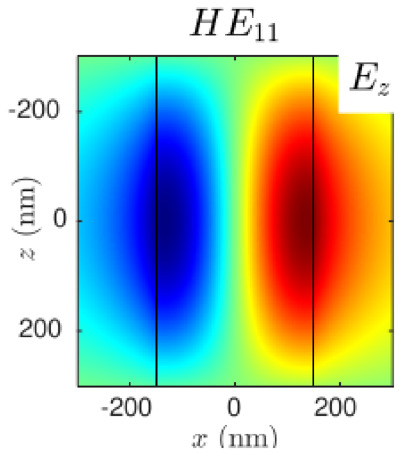
Approximate Modes in Dielectric Discs



Tracking Modes Across Dielectric Discs







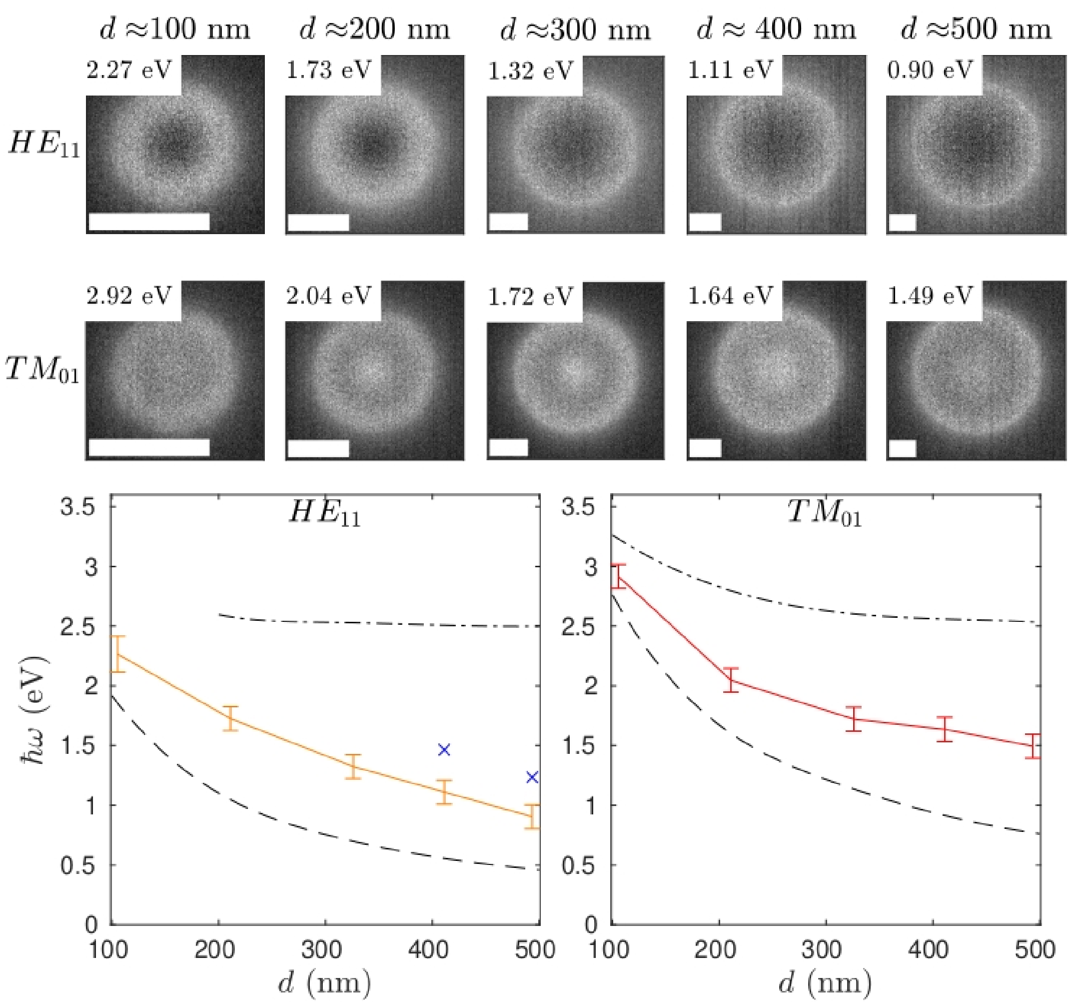
Different modes with similar qualities are imaged at different beam energies

$$k_z = \omega_{\ell mn}/v$$

$$k_z = \frac{2\pi}{100\,\text{nm}}$$

Waveguide model gives us a “lower bound” on the mode energy, and thin disc approximation gives us an “upper bound”

Tracking Modes Across Discs



Conclusions, Thanks

- Spatial features of electrodynamic modes in dielectric particles can now be imaged using STEM-EELS
- Key features of these modes can be captured by modeling them as the Cherenkov excitation of waveguide modes

Dr. Duncan T.L. Alexander

Dr. Christian Dwyer