Beats, Overtones, and Musical Temperament

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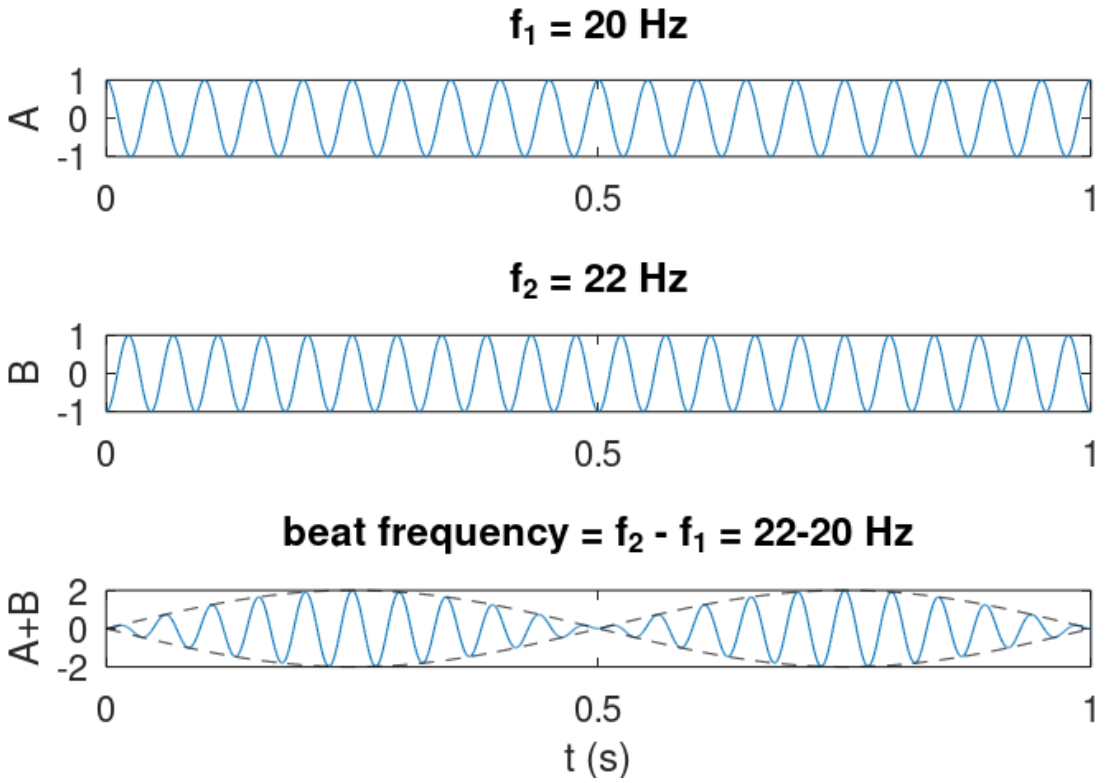
Outline

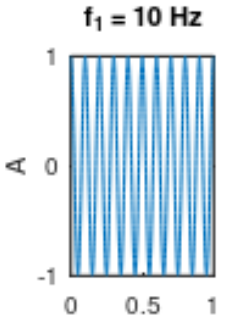
- Review: Acoustic Beats and Interference
- Review: Harmonic Series
- Suggested tweak: Connect the AB and HS
  - Beat-free interference with “just” intonation
  - Harmonic “temperament” and history
- Distinguishing Between Notation and Reality

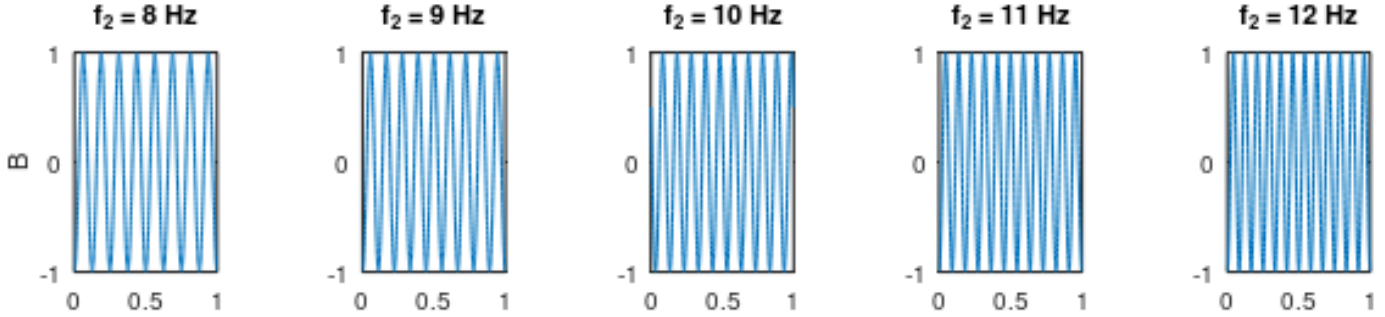
Interference and Beats

- Coherent interference between sound waves of differing frequencies causes variations in sound intensity
- The “beat frequency” here is equal to the difference of the original frequencies

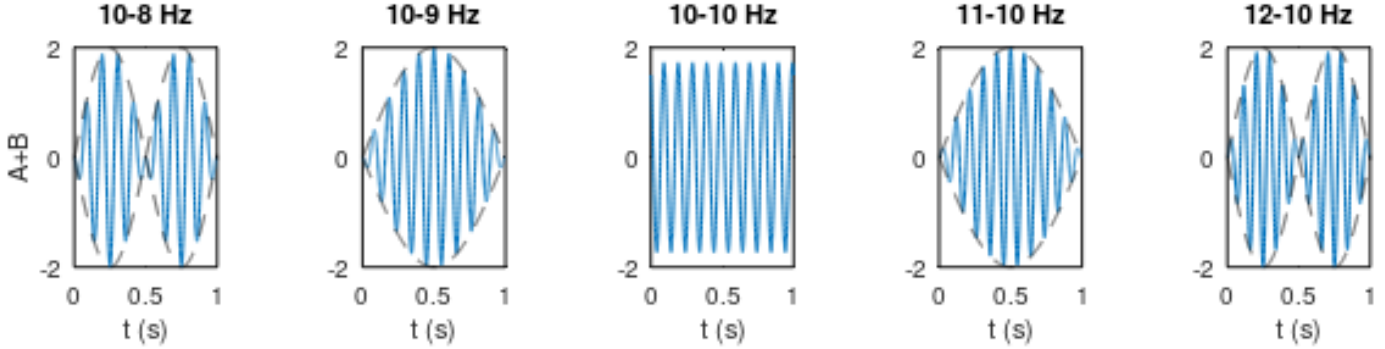


Discovery by Experiment!

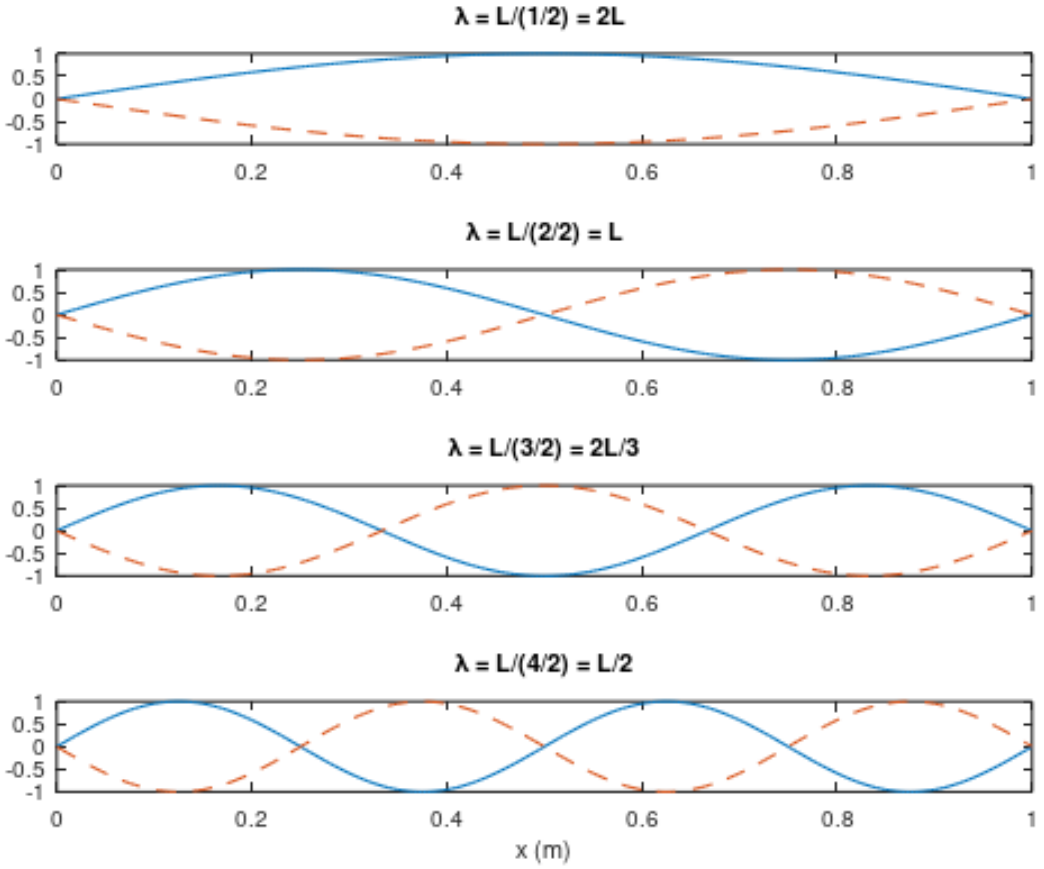




Beat frequency =
$|f_2 - f_1|$



Harmonic Series



f_1 = 100 Hz

One octave → 2 x f_1

$f_2 = 2f_1 = 200$ Hz

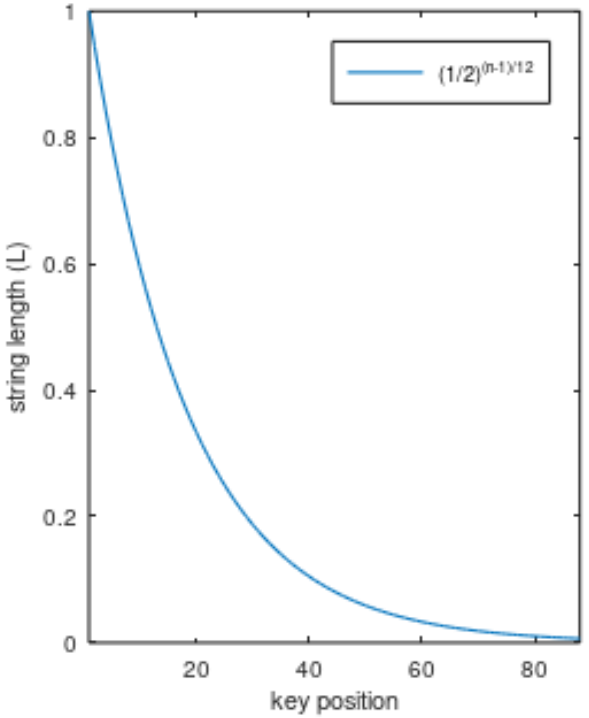
$f_2 = 3f_1 = 300$ Hz

$f_2 = 4f_1 = 400$ Hz

Two octaves → 4 x f_1

Real vs. Idealized Pianos

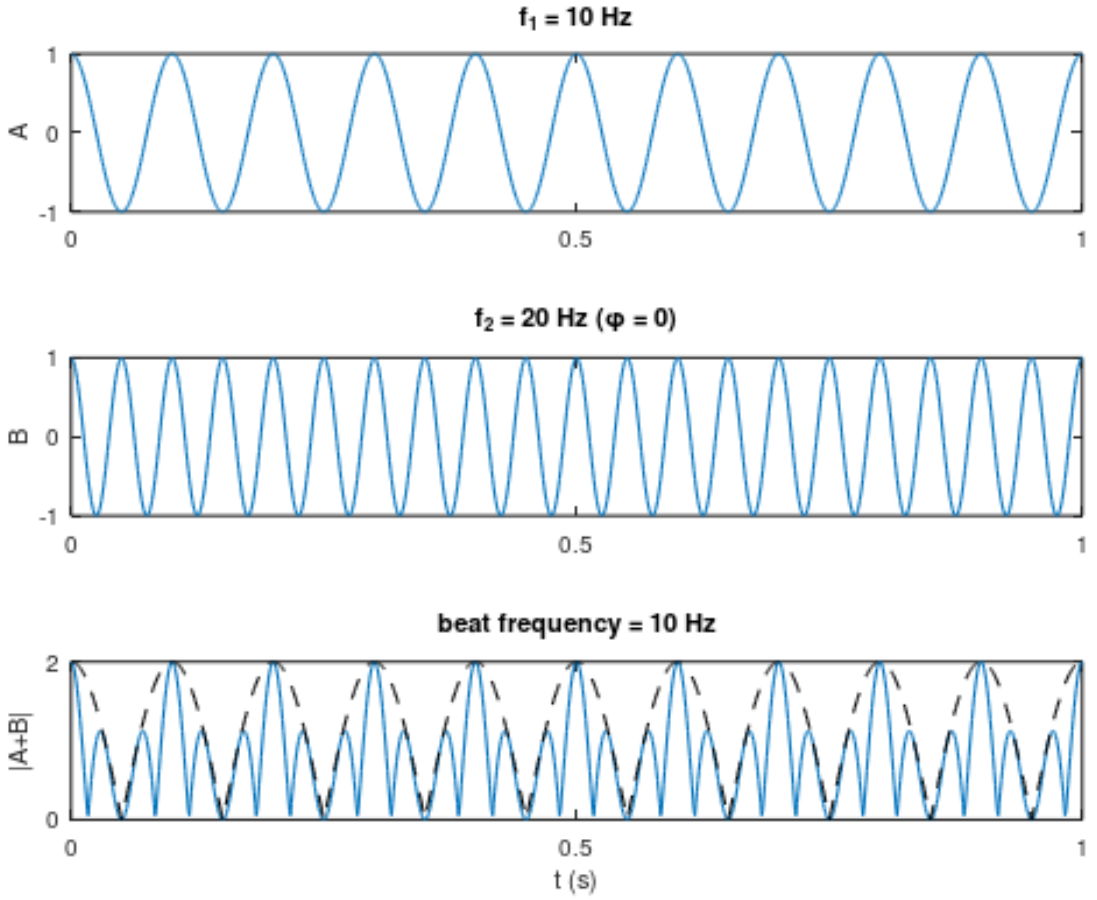
- Consider an idealized piano with strings of all the same thickness and tension. What is its shape?



$$L\,(1/2)^{n/12}$$

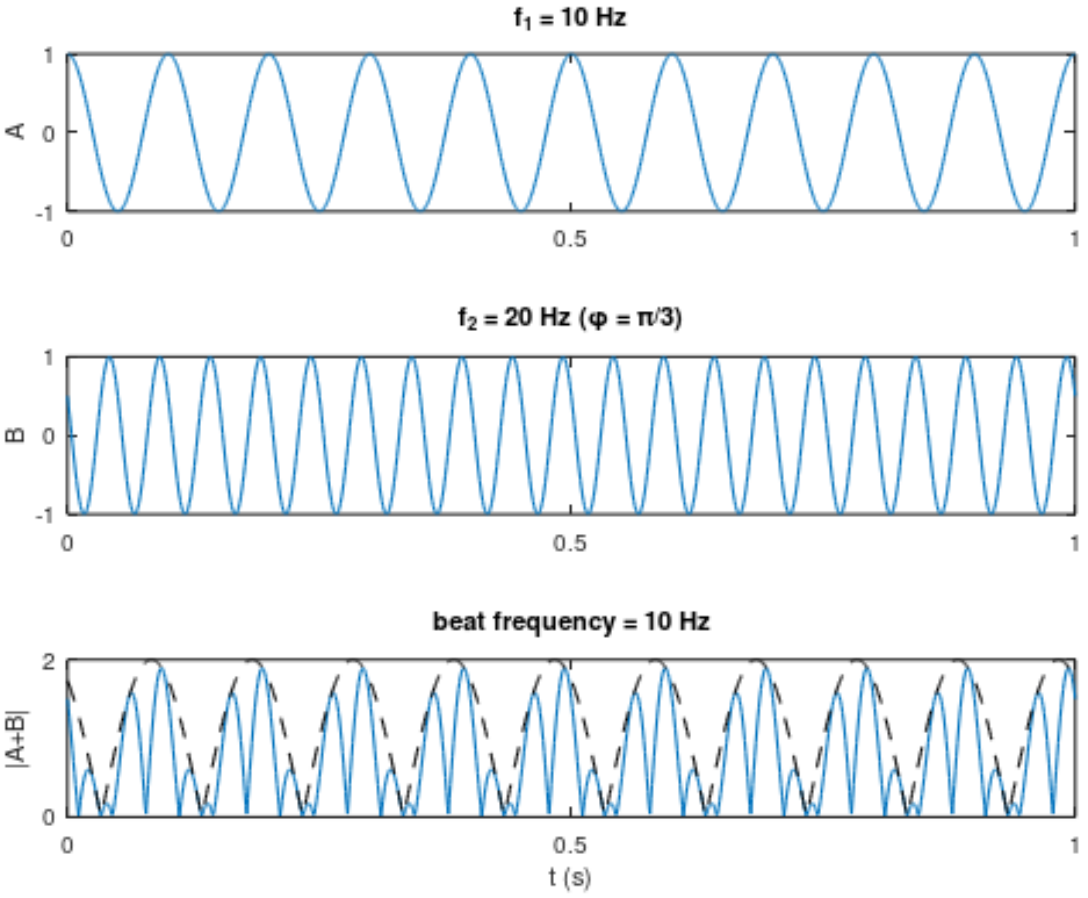
Harmonics and Beat-free Interference

- The “beat frequencies” for summed harmonics ...are still harmonics



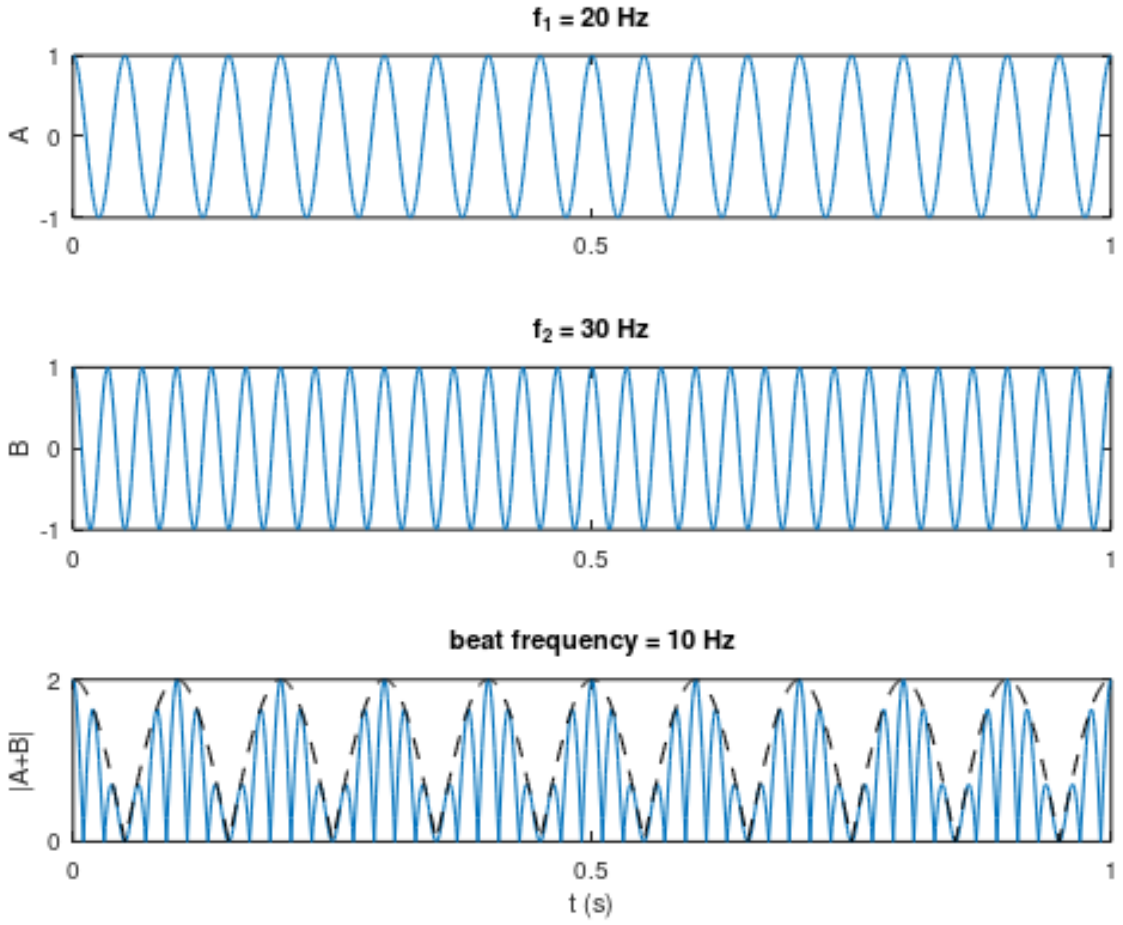
Harmonics and Beat-free Interference

- The “beat frequencies” for summed harmonics ...are still harmonics
- This is true regardless of the phase of the input sound waves…

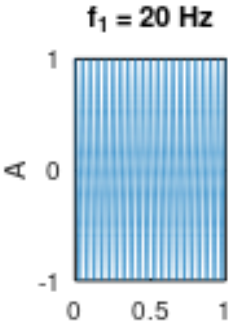


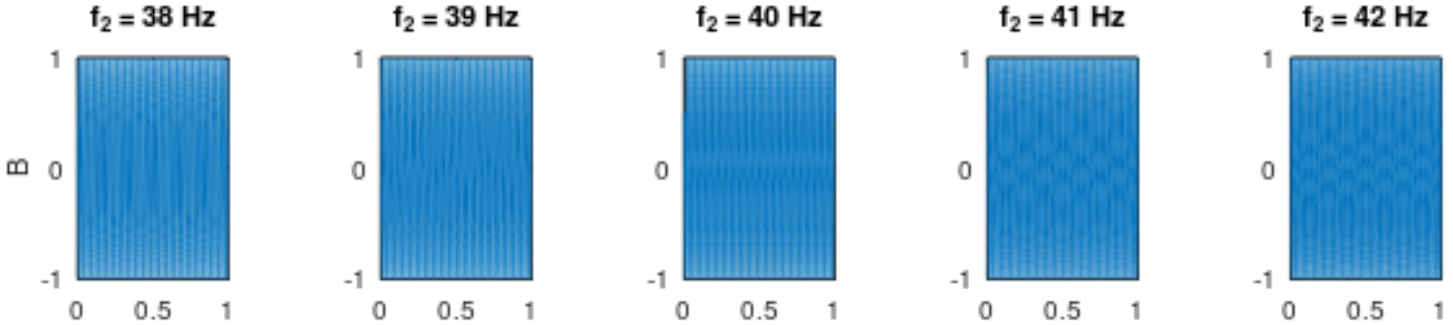
Harmonics and Beat-free Interference

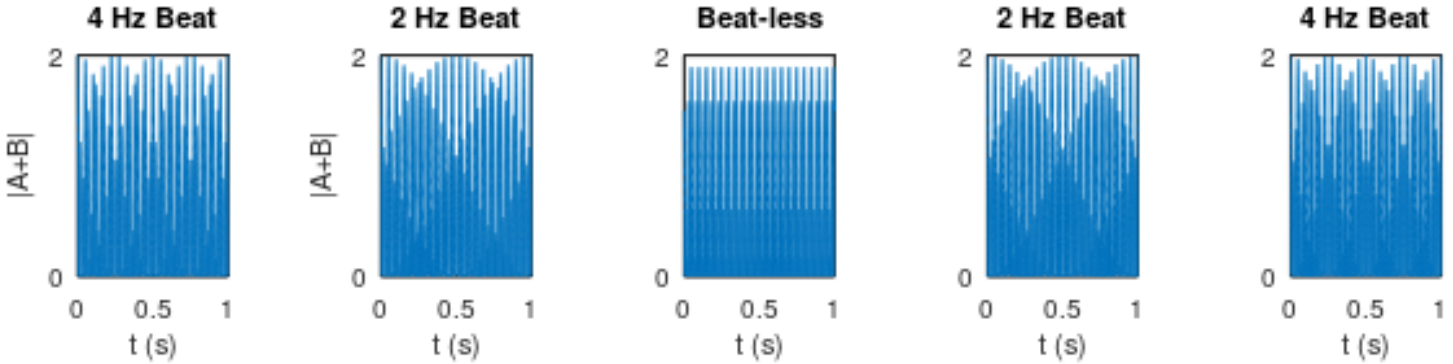
- The “beat frequencies” for summed harmonics ...are still harmonics
- This is true regardless of the phase of the input sound waves…
- ...or of which members of the harmonic series one might play together



Discovery by Experiment?

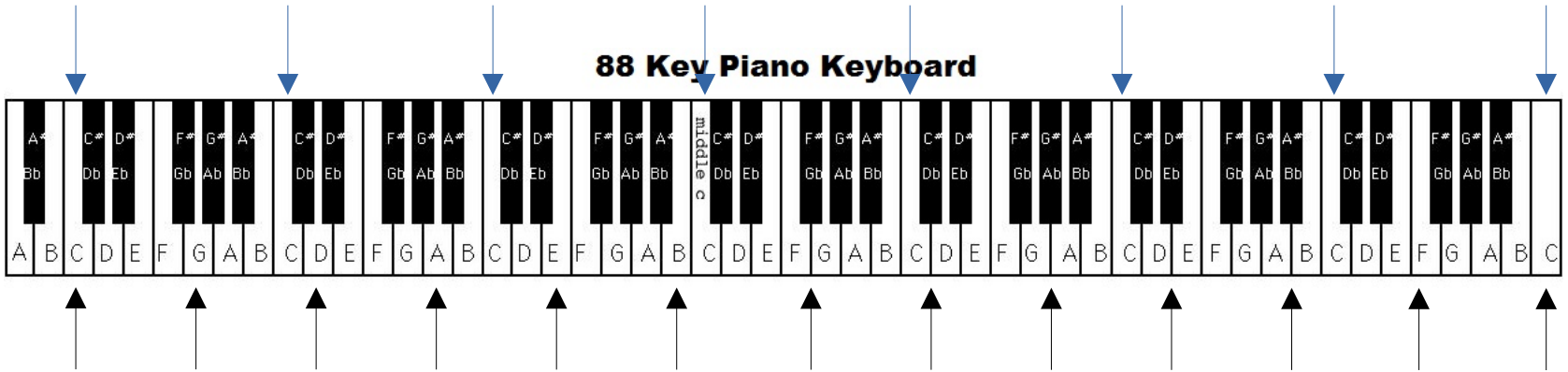






The Problem of Temperament

- On a piano whose octave has 12 half-steps, the “circle of fifths” closes after 12 perfect fifths and 7 octaves



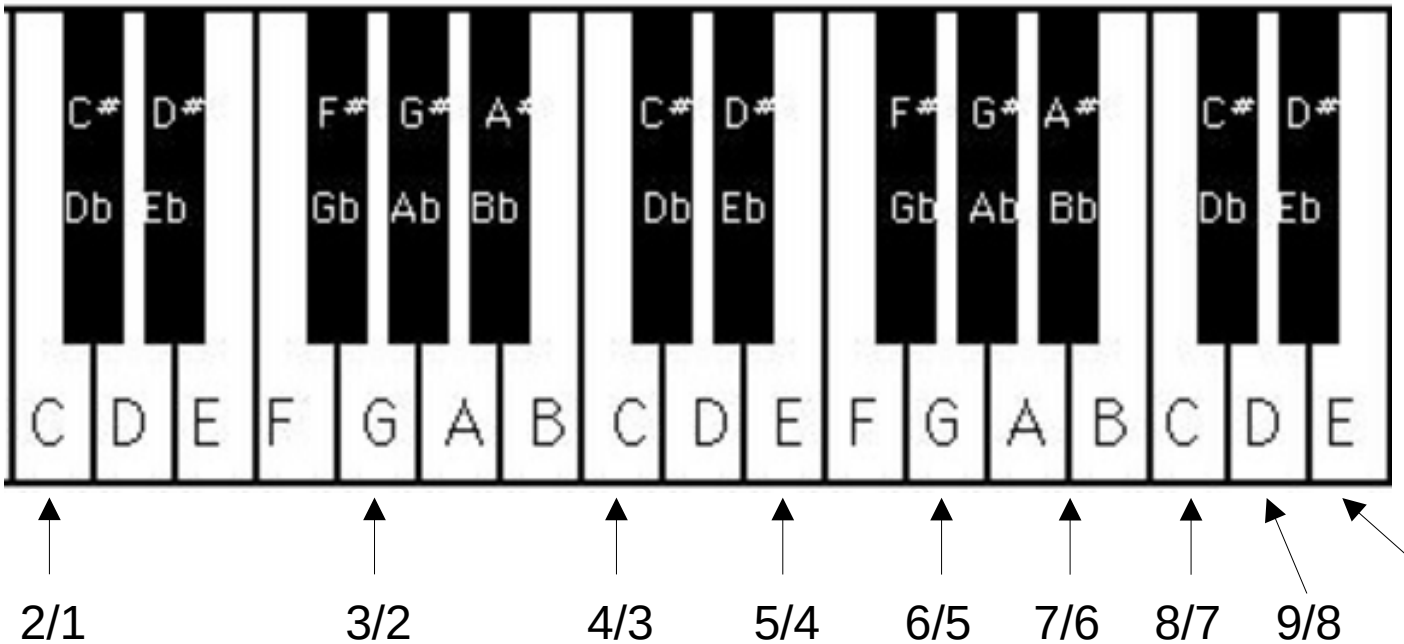
- This works by making the fifths just slightly small:

$$(2/1)^{7/12} \approx (3/2)$$

$$1.4983... \approx 1.5$$

Overloaded Musical Notation

- The overtone series is doubly confusing because the approximation of equal half-steps corrupts the “natural” overtone series



Minor 3rd:
6/5 or 7/6?

Minor 2nd:
8/7 or 9/8 or 10/9?

Conclusion

- Beat frequencies and natural harmonics are tied together via the beat frequencies of interfering waves from the same harmonic series
- Musicians at times denote similar yet distinct harmonic intervals with undifferentiated notation – yet physicists must admit similar sins

Useful Resources

- A website: “Twelve-Tone Musical Scale”: thinkzone.wlonk.com/Music/12Tone.htm
- An argument: *How Equal Temperament Ruined Harmony* by Ross W. Duffin
- A college-level textbook: *The Arithmetic of Listening* by Kyle Gann